Test 1 Review  Math1312  Section

Is not Multiple choice!

When: Monday, Sep 20
Where: In our regular classroom.
What to bring: Picture ID, Pencil, eraser, Compass and protractor.
What is covered: 1.1 – 2.3.

Vocabulary: You need to know what these terms mean, but you do not need to memorize the definitions word by word.
1.1: Statement, open statement, conditional statement, conjunction, disjunction, negation, the law of detachment.
1.2: Line segment, congruent line segments, collinear points, midpoint, bisector, straight angle, right angle, perpendicular lines.
1.3: Segment – Addition Postulate, Angle-Addition Postulate
1.4: Acute angle, right angle, straight angle, obtuse angle, reflex angle, congruent angles, vertical angles, complementary angles, supplementary angles.
1.5: Addition Property, Subtraction Property, Multiplication Property, Division Property, Substitution Property, Distributive Property, Transitive Property.
1.6: Reflexive Property, Symmetric Property, Transitive Property.
1.7: Hypothesis and Conclusion of a Conditional Statement, converse of a conditional statement.
2.1: Transversal, interior-exterior angles, alternate interior angles, alternate exterior angles, corresponding angles, consecutive interior angles.
2.2: Conditional statement and its inverse, converse, contrapositive, the law of negative inference.

Postulates and Theorems: You need to know these as facts. You do not need to memorize them with their numbers, but know what they say. You need to know “Segment – Addition Postulate, Angle-Addition Postulate” by name since you may need to use them in proofs.

Constructions: You need to know the constructions. I may ask one construction. Please do not forget to bring your protractor and compass to the test.

Proofs: Study the examples which are about doing proofs (both in my class notes and in the text book). You do not need to memorize them. I will not ask difficult proofs, but you need to know what to write as a "reason", the best way to learn this is by going over the examples.

How to study: Study the class notes and try to do as many exercises as you can from the suggested homework list. These terms are new to most of you. You will get used to them as you solve the exercises.
Some examples: Here I provided some examples for you. This is not a complete list, studying only these examples is not enough!

1) a) Conditional Statement: If all sides of a rectangle are equal, then it is a square.

Hypothesis: All sides of a rectangle are equal

Conclusion: Rectangle is a square

Converse: \( Q \Rightarrow P \): If a rectangle is a square, then all sides of a rectangle are equal.

Inverse: \( \sim P \Rightarrow \sim Q \): If not all sides of a rectangle are equal, then it is not a square.

Contrapositive: \( \sim Q \Rightarrow \sim P \): If a rectangle is not a square, then not all sides of a rectangle are equal.

(b) If \( x > 2 \), then \( x \neq 0 \)

Hypothesis: \( x > 2 \)

Conclusion: \( x \neq 0 \)

Converse: \( Q \Rightarrow P \): If \( x \neq 0 \), then \( x > 2 \).

Inverse: \( \sim P \Rightarrow \sim Q \): If \( x \leq 2 \), then \( x = 0 \)

Contrapositive: \( \sim Q \Rightarrow \sim P \): If \( x = 0 \), then \( x \leq 0 \)

c) Two angles are complementary if the sum of their measures is \( 90^\circ \).

Hypothesis: The sum of measures of two angles is \( 90^\circ \)

Conclusion: Two angles are complementary

Converse: If two angles are complementary, then the sum of their measures is \( 90^\circ \).

Inverse: If the sum of measures of two angles is not equal to \( 90^\circ \), then they are not complementary

Contrapositive: If two angles are not complementary, then the sum of their measures is not \( 90^\circ \).
2) \[ \frac{4x-3}{2x+1} \]

\[ BC = 2x+1, \ AB = 4x - 3. \] If AC is 22, then what is AB? What is BC?

\[ \begin{align*}
\text{AB} + \text{BC} &= \text{AC} \\
4x-3 + 2x+1 &= 22 \\
6x-2 &= 22 \\
6x &= 24 \\
x &= 4
\end{align*} \]

\[ \text{AB} = 4(4) - 3 = 16 - 3 = 13 \\
\text{BC} = 2(4) + 1 = 8 + 1 = 9 \]

3)

a) If \( m\angle 1 = 2x \), \( m\angle 2 = 3x - 10 \), and \( m\angle KLM = 100^\circ \), then what is \( x \)?

\[ m\angle 1 + m\angle 2 = m\angle KLM \]

\[ 2x + 3x - 10 = 100 \]

\[ 5x = 110 \]

\[ x = 22 \]

b) If \( \overrightarrow{LN} \) is a bisector and if \( m\angle 1 = 2x + 10 \), \( m\angle 2 = 3x - 45 \), then what is \( m\angle KLM \)?

\[ m\angle 1 = m\angle 2 \]

\[ 2x+10 = 3x-45 \]

\[ 55 = x \]

\[ m\angle KLM = m\angle 1 + m\angle 2 = 2(55) + 10 + 3(55) - 45 = 240^\circ \]
4) Find the measures of all the angles 1 - 7.

\[ \angle 1 = 180^\circ - 55^\circ = 125^\circ \]
\[ \angle 2 = 125^\circ \text{ (vertical to } \angle 1) \]
\[ \angle 3 = 180^\circ - 100^\circ = 80^\circ \]
\[ \angle 4 = 100^\circ \text{ (vertical to } 100^\circ) \]
\[ \angle 5 = 55^\circ \text{ (vertical to } 55^\circ) \]

**Hint:** \( \angle 3 + \angle 5 + \angle 6 = 180^\circ \)

\[ \angle 6 = 180^\circ - (\angle 3 + \angle 5) = \]
\[ = 180^\circ - (80 + 55) = 45^\circ \]
\[ \angle 7 = 45^\circ \text{ (vertical to } \angle 6) \]

5)
Given: \( 3x + 2 = 4 + 5x \)
Prove: \( x = -1 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( 3x + 2 = 4 + 5x )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( 3x + 2 - 4 = 4 - 4 + 5x )</td>
<td>2. Subtraction Property of Equality</td>
</tr>
<tr>
<td>3. ( 3x - 2 = 5x )</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. ( 3x - 3x - 2 = 5x - 3x )</td>
<td>4. Subtraction Property of Equality</td>
</tr>
<tr>
<td>5. ( -2 = 2x )</td>
<td>5. Substitution</td>
</tr>
<tr>
<td>6. ( \frac{1}{2}(-2) = \left(\frac{1}{2}\right)2x )</td>
<td>6. Multiplication/Division Property of Eq.</td>
</tr>
<tr>
<td>7. ( -1 = x )</td>
<td>7. Substitution</td>
</tr>
<tr>
<td>8. ( x = -1 )</td>
<td>8. Symmetry prop. i.e. ( a = b \Rightarrow b = a )</td>
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</table>
6) Given: B is the midpoint of the line \( \overline{AC} \)
Prove: \( AB = \frac{AC}{2} \)

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<tr>
<td>1. B is the midpoint of ( \overline{AC} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB = BC )</td>
<td>2. Def. of a midpoint</td>
</tr>
<tr>
<td>3. ( AB + BC = AC )</td>
<td>3. Segment Addition Postulate</td>
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<tr>
<td>4. ( AB + AB = AC )</td>
<td>4. Substitution (2, to 3)</td>
</tr>
<tr>
<td>5. ( 2(AB) = AC )</td>
<td>5. Substitution</td>
</tr>
<tr>
<td>6. ( AB = \frac{AC}{2} )</td>
<td>6. Division Property of Equality</td>
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7) Check if the following relations have the reflexive, symmetric, transitive properties.

a) relation: “greater than” \((x > y)\)
   - reflexive: \( x > x \) ? False \((\text{Ex}: 1 \neq 1)\)
   - symmetric: If \( x > y \), then \( y > x \) ? False \((5 \neq 10, \text{but } 10 \neq 5)\)
   - transitive: If \( x > y \) and \( y > z \), then \( x > z \)? True \((\text{Ex}: 5 > 1 \text{ and } 1 > 0, \text{then } 5 > 0)\)

b) relation: “complementary to” (angles)
   - reflexive: \( \angle 1 \) is comp. to \( \angle 1 \) ? False \((\angle 1 \text{ is NOT complementary to itself})\)
   - symmetric: If \( \angle 1 \) is comp. to \( \angle 2 \) and \( \angle 2 \) is comp. to \( \angle 3 \), then \( \angle 1 \) is comp. to \( \angle 3 \)? True
   - transitive: If \( \angle 1 \) is comp. to \( \angle 2 \) and \( \angle 2 \) is comp. to \( \angle 3 \), then \( \angle 1 \) is comp. to \( \angle 3 \)? True \((\text{Ex}: m\angle 1 = 50, m\angle 2 = 40, m\angle 3 = 50)\)

(\( \angle 1, \angle 2, \angle 3 \text{ are NOT comp}\))

c) relation: “congruent” (line segments)
   - reflexive: If \( \overline{AB} \cong \overline{CD} \) then \( \overline{CD} \cong \overline{AB} \) ? True \((\text{Ex}: m\overline{L1} = 50)\)
   - symmetric: If \( \overline{AB} \cong \overline{CD} \) then \( \overline{CD} \cong \overline{AB} \) ? True \((\text{Ex}: m\overline{L2} = 40)\)
   - transitive: If \( \overline{AB} \cong \overline{CD} \) and \( \overline{CD} \cong \overline{PQ} \), then \( \overline{AB} \cong \overline{PQ} \) True
If \( m \angle 3 = 6x + 10 \) and \( m \angle 4 = 2x + 10 \), solve for \( x \) and find the measures of angles 3 and 4.

\[
\begin{align*}
\text{m \angle 3 + m \angle 4} & = 180^\circ \\
6x + 10 + 2x + 10 & = 180^\circ \\
8x + 20 & = 180 \\
8x & = 160 \\
x & = 20
\end{align*}
\]

\[
\begin{align*}
m \angle 3 & = 6(20) + 10 = 120 + 10 = \frac{130^\circ}{\text{ }} \\
m \angle 4 & = 2(20) + 10 = 40 + 10 = 50^\circ \\
\end{align*}
\]

If \( a \parallel b \) and \( m \angle 5 = 95^\circ, m \angle 3 = 105^\circ \), then find the measures of all the other angles.

\[
\begin{align*}
m \angle 1 & = 105^\circ \ (\text{vertical to } \angle 3) \\
m \angle 2 & = 180 - 105 = 75^\circ \ (\text{supp. to } \angle 3) \\
m \angle 4 & = 75^\circ \ (\text{vertical to } \angle 2) \\
m \angle 6 & = 105^\circ \ (\text{alt. int. to } \angle 3) \\
m \angle 7 & = 105^\circ \ (\text{vertical to } \angle 8) \\
m \angle 8 & = 180 - 105 = 75^\circ \ (\text{supp. to } \angle 8) \\
m \angle 6 & = 180 - 95 = 85^\circ \ (\text{supp. to } \angle 7)
\end{align*}
\]
10) \[ \overline{AB} || \overline{CD} \]

\[ \overrightarrow{FE} \text{ bisects } \angle AFG \]

If \( m\angle 4 = 2x - 20 \), \( m\angle 5 = x + 30 \), find the measures of angles 1-13.

\( \angle 4 \) and \( \angle 5 \) are int. alt. \( \angle 5 \)

\[ 2x - 20 = x + 30 \]

\[ x = 50 \]

\[ m\angle 4 = 2(50) - 20 = 80^\circ \]

\[ m\angle 5 = 50 + 30 = 80^\circ \]

See class notes for a similar problem!

11) Give an indirect proof:

Given: \( \angle 3 \neq \angle 7 \)

Prove: \( a \parallel b \)

Suppose that \( a \parallel b \)

Then \( \angle 3 \cong \angle 7 \) (corresponding angles)

But \( \angle 3 \neq \angle 7 \).

Assumption must be false!

Hence \( a \parallel b \).
12) a) Find the value of $x$ and the measure of each angle that will make $p \parallel q$

$$4x + 60 = 14x - 60$$
$$120 = 10x$$
$$x = 12$$

b) Find the value of $x$ and $y$ and that will make $p \parallel q$

$$4x + 50 = 110$$
$$4x = 60$$
$$x = 15$$

$$2y - 50 = 70$$
$$2y = 120$$
$$y = 60$$
13) BC \parallel DE. If angles 1 and 2 are complementary, and if $m\angle 3 = 40^\circ$, find $m\angle 1$ and $m\angle 2$.

\[ m\angle 1 + m\angle 2 = 90^\circ \]
\[ m\angle 1 + 40^\circ = 90^\circ \]
\[ m\angle 1 = 50^\circ \]

BC \parallel DE

$m\angle 2 = m\angle 3$

$m\angle 2 = 40^\circ$