Math 1312  
Section 5.5  
Special Right Triangles

**Note:** Triangles in this section are always right triangles!

**45-45-90 Triangles**

**Theorem 1:** In a triangle whose angles measure $45^\circ$, $45^\circ$, and $90^\circ$, the hypotenuse has a length equal to the product of $\sqrt{2}$ and the length of either leg.

The ratio of the sides of a 45-45-90 triangle are: $x : x : x\sqrt{2}$.

<table>
<thead>
<tr>
<th>if this measure is given:</th>
<th>and you want this measure:</th>
<th>then do this:</th>
</tr>
</thead>
<tbody>
<tr>
<td>the leg</td>
<td>hypotenuse</td>
<td>multiply the leg by $\sqrt{2}$</td>
</tr>
<tr>
<td>hypotenuse</td>
<td>the leg</td>
<td>divide hypotenuse by $\sqrt{2}$</td>
</tr>
</tbody>
</table>

**Note:** The legs of a 45-45-90 are the same measure!

$$\text{leg} = \frac{\text{hypotenuse}}{\sqrt{2}} \quad \text{hypotenuse} = \text{leg}\sqrt{2}$$

**30-60-90 Triangles**

**Theorem 2:** In a triangle whose angles measure $30^\circ$, $60^\circ$, and $90^\circ$, the hypotenuse has a length equal to twice the length of the shorter leg, and the length of the longer leg is the product of $\sqrt{3}$ and the length of the shorter leg.

The ratio of the sides of a 30-60-90 triangle are: $x : x\sqrt{3} : 2x$.

**Note:** The short leg is always opposite the $30^\circ$ angle!

It is best to find the measure of the short leg first (that is if it is not given).
### 30-60-90

<table>
<thead>
<tr>
<th>if this measure is given:</th>
<th>and you want this measure:</th>
<th>then do this:</th>
</tr>
</thead>
<tbody>
<tr>
<td>short leg</td>
<td>hypotenuse</td>
<td>multiply short leg by 2</td>
</tr>
<tr>
<td>short leg</td>
<td>long leg</td>
<td>multiply short leg by $\sqrt{3}$</td>
</tr>
<tr>
<td>long leg</td>
<td>short leg</td>
<td>divide long leg by $\sqrt{3}$</td>
</tr>
<tr>
<td>hypotenuse</td>
<td>short leg</td>
<td>divide hypotenuse by 2</td>
</tr>
</tbody>
</table>

\[
\text{short \_leg} = \frac{\text{hypotenuse}}{2} \quad \text{short \_leg} = \frac{\text{long \_leg}}{\sqrt{3}}
\]

\[
\text{hypotenuse} = 2(\text{short \_leg}) \quad \text{long \_leg} = \sqrt{3}(\text{short \_leg})
\]

**Example 1:** Find the altitude of the following equilateral triangle.

![Equilateral triangle](image)
Example 2: Find the altitude of the following equilateral triangle.

Example 3: Find the length of the side of the following equilateral triangle.
Example 4: Find a and b.

Example 5: Find c and d.