Sample Midterm 1 of Honors Calculus.

There are 25 points in total. Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work. Points will not be awarded if working is not shown.

1. (5 points) State without proof the following limits:

   (i) \( \lim_{x \to 0} \left( \frac{\sin 3x}{x} \right)^4 \)

   (ii) \( \lim_{x \to \infty} \frac{x^2 + \sqrt{x}}{x^2 + x^{3/2} + 1} \)

   (iii) \( \lim_{x \to 0} e^x \sin(x) \)

   (iv) \( \lim_{x \to \infty} e^{-\sqrt{x}} x^2 \)

2. (5 points) A particle with coordinates \((x(t), y(t))\) is constrained to move on the ellipse

   \[ \frac{y^2}{4} + x^2 = 1 \]

   Find the points on the ellipse where \( \frac{dx}{dt} = \frac{dy}{dt} \). Hint: This is a related rates problem.

3. (5 points) Suppose \( x \geq 0, y \geq 0 \) and \( x + y = 1 \). Find the maximum value of

   \[ -x \ln(x) - y \ln(y) \]

4. (5 points) Draw the graph of a differentiable function on \([0, \infty)\) which has all the following properties:

   • \( f \) has a global minimum at \( x = 0 \) and \( f(0) = 0 \)
   • \( f \) is concave up on \([0, 1)\) with a point of inflection at \( x = 1 \)
   • \( f \) has a global maximum at \( x = 2 \) and \( f(2) = 4 \)
   • \( f \) is increasing on \([0, 2)\) and decreasing on \((2, \infty)\)
   • \( \lim_{x \to \infty} f(x) = 1 \)

5. (5 points) The rate of radioactive decay of cesium of mass \( P \) with respect to time is proportional to the mass of cesium remaining.

   (a) Write down an equation expressing the statement above.

   (b) Suppose that 200 grams of cesium has decayed to 50 grams over the period of 4 months, at which time another 10 grams is added. Find the total amount of the 60 grams of Cesium remaining 5 months after the 10 grams was added.