Lecture 10
Section 3.9 Differentials; Newton Approximation
Section 4.1 Mean-Value Theorem

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Test 1

- Test 1 - updated due to ike.
- October 7-9 in CASA
- Loggin to CourseWare to reserve your time to take the exam.
Online Quizzes

- Online Quizzes are available on CourseWare.
- The due dates for Quizzes have been extended.
Review for Test 1 by by Prof. Morgan.

Tonight 8:00 - 10:00pm in 100 SEC
Quiz 1

\[
\lim_{x \to 0} \frac{\sin(7x)}{\sin(5x)}
\]

a. 1

b. 1/3

c. 7/5

d. 2

e. None of these
Where is \( f(x) = \frac{x-1}{x^2-1} \) continuous?

- a. everywhere
- b. \( x = 1, -1 \)
- c. \( x = 1 \)
- d. everywhere except \( x = -1 \)
- e. None of these
Find the slope of the tangent line to the graph of $f(x) = x^2 + 3x$ at $x = 1$.

a. 4

b. 5

c. 6

d. 7

e. None of these
Differentials

- increment: $\Delta f = f(x + h) - f(x)$
- differential: $df = f'(x)h$

$\Delta f \approx df$

in the sense that $\frac{\Delta f - df}{h}$ tends to 0 as $h \to 0$. 
Newton Method

Let the number $c$ be a solution (root) of an equation $f(x) = 0$. The Newton-Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

generates a sequence of approximations $x_1, x_2, \ldots, x_n, \ldots$ that will “converge” to the root $c$
Convexity Conditions for the Convergence

looping or divergent

\[ f(x)f''(x) > 0 \]
Example: Estimate $\sqrt{3}$

The number $\sqrt{3}$ is a root of the equation $x^2 - 3 = 0$. Estimate $\sqrt{3}$ by applying the Newton method to the function $f(x) = x^2 - 3$ starting at $x_1 = 2$:

$$x_{n+1} = x_n - \frac{x_n^2 - 3}{2x_n} = \frac{x_n^2 + 3}{2x_n}.$$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x_n$</th>
<th>$x_{n+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.75000</td>
</tr>
<tr>
<td>2</td>
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<td>1.73214</td>
</tr>
<tr>
<td>3</td>
<td>1.73214</td>
<td>1.73205</td>
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The Mean-Value Theorem

If $f$ is differentiable on the open interval $(a, b)$ and continuous on the closed interval $[a, b]$, then there is at least one number $c$ in $(a, b)$ for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently

$$f(b) - f(a) = f'(c)(b - a).$$
Theorem

Let \( f \) be differentiable on the open interval \((a, b)\) and continuous on the closed interval \([a, b]\). If \( f(a) = f(b) = 0 \), then there is at least one number \( c \) in \((a, b)\) at which

\[ f'(c) = 0. \]