Test 1

- The written questions on Test 1 are graded and appear as a separate column in your CourseWare gradebook.
- You have to add the two columns “Test 1” and “FR1” to get your total score on the exam.
- The average in this class was 65.5!!! (Others 77.63, 75.01, 70.95)

Grade Information

- 90% and above - A
- at least 80% and below 90% - B
- at least 70% and below 80% - C
- at least 60% and below 70% - D
- below 60% - F

Grade Information

- 300 points determined by exams 1, 2 and 3
- 100 points determined by lab work, written quizzes, homework, daily grades and online quizzes.
- 200 points determined by the final exam
- 600 points total

Weekly Online Quizzes

- Online quizzes are given most weeks.
- You can attempt these quizzes as many times as you like until they expire.
- The highest grade will be used for your score.
- If you fail to reach 70% during three weeks of the semester, I have the option to drop you from the course!!!
Quiz 1
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical value at 2 or state that the value is not a critical value.

a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these

Quiz 2
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical value at 3 or state that the value is not a critical
1 Section 4.5 Some Max-Min Problems (Cont.)

Example 3

value.

a. local maximum
b. local minimum
c. neither
d. not a critical value
e. None of these
Example 3  A window in the shape of a rectangle capped by a semicircle is to have perimeter $p$. Choose the radius of the semicircular part so that the window admits the greatest amount of light.

Example 4

Example 4  A state highway department plans to construct a new road between towns $A$ and $B$. Town $A$ lies on an abandoned road that runs east-west. Town $B$ is 3 miles north of the point on that road that is 5 miles east of $A$. The engineering division proposes that the road be constructed by restoring a section of the old road from $A$ up to a point $P$ and joining it to a new road that connects $P$ and $B$. If the cost of restoring the old road is $200,000 per mile and the cost of the new road is $400,000 per mile, how much of the old road should be restored in order to minimize the department's costs?

Example 5
Example 5  (The angle of incidence equals the angle of reflection.) Figure 4.5.6 depicts light from a point $A$ reflected to a point $B$ by a mirror. Two angles have been marked: the angle of incidence, $\theta_1$, and the angle of reflection, $\theta_r$. Experiment shows that $\theta_1 = \theta_r$. Derive this result by postulating that the light that travels from $A$ to the mirror and then to $B$ follows the shortest possible path.

Example 6  A manufacturing plant has a capacity of 25 articles per week. Experience has shown that $n$ articles per week can be sold at a price of $p$ dollars each where $p = 110 - 2n$ and the cost of producing $n$ articles is $600 + 10n + n^2$ dollars. How many articles should be made each week to give the largest profit?

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2  Section 4.6 Concavity and Points of Inflection

Concavity and Points of Inflection

Definition 1.  • The graph of $f$ is **concave up** on $I$ if $f'$ **increases** on $I$.  


• The graph of $f$ is **concave down** on $I$ if $f'$ decreases on $I$.

• Points that join arcs of **opposite concavity** are **points of inflection**.

**Example 1**

- Determine the intervals on which $f$ increases and the intervals on which $f$ decreases.
- Determine the intervals on which the graph of $f$ is concave up and the intervals on which the graph of $f$ is concave down.
- Give the $x$-coordinates of the points of inflection.

**Example 2**

- Determine the intervals on which $f$ increases and the intervals on which $f$ decreases.
- Determine the intervals on which the graph of $f$ is concave up and the intervals on which the graph of $f$ is concave down.
- Give the $x$-coordinates of the points of inflection.
Second-Derivative Test

**Theorem 2.**
- If $f''(x) > 0$ for all $x$ in $I$, then $f'$ increases on $I$, and the graph of $f$ is concave up.
- If $f''(x) < 0$ for all $x$ in $I$, then $f'$ decreases on $I$, and the graph of $f$ is concave down.
- If the point $(c, f(c))$ is a point of inflection, then either $f''(c) = 0$ or $f'(c)$ does not exist.

**Example 3**
Determine concavity and find the points of inflection of the graph of \( f(x) = x^3 - 6x^2 + 9x + 1 \).

\[ f'(x) = 3x^2 - 12x + 9, \quad f''(x) = 6x - 12. \]
Example 4

- Determine concavity and find the points of inflection of the graph of $f(x) = x + \cos x$, $x \in [0, 2\pi]$.

$f'(x) = 1 - \sin x$, $f''(x) = -\cos x$.

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<tr>
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<td>concave up</td>
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<tr>
<td>$\infty$</td>
<td>concave down</td>
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Graph showing points of inflection at $(\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, \frac{3\pi}{2})$. Graph also shows $f(x) = x + \cos x$.