Section 4.7 Vertical and Horizontal Asymptotes; Vertical Tangents and Cusps

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Test 2

- Test 2: November 1-4 in CASA
- Loggin to CourseWare to reserve your time to take the exam.
Review for Test 2 by the College Success Program.

Friday, October 24 2:30–3:30pm in the basement of the library by the C-site.
Grade Information

- 300 points determined by exams 1, 2 and 3
- 100 points determined by lab work, written quizzes, homework, daily grades and online quizzes.
- 200 points determined by the final exam
- 600 points total
More Grade Information

- 90% and above - A
- at least 80% and below 90% - B
- at least 70% and below 80% - C
- at least 60% and below 70% - D
- below 60% - F
Online Quizzes are available on CourseWare.

If you fail to reach 70% during three weeks of the semester, I have the option to drop you from the course!!!
Tuesday, November 4, 2008

Last day to drop a course or withdraw with a “W” (must be by 5 pm)
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Give the number of critical values of $f$.

a. 2  
b. 3  
c. 4  
d. 5  
e. None of these
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Give the number of intervals of increase of $f$.

a. 1  
b. 2  
c. 3  
d. 4  
e. None of these
Assume the domain of \( f \) is all real numbers. The graph of \( f'(x) \) is shown below. Give the number of intervals of decrease of \( f \).

a. 1  

b. 2  

c. 3  

d. 4  

e. None of these
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Classify the smallest critical number of $f$

a. local maximum  
b. local minimum  
c. neither
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Classify the critical number of $f$ between 0 and 2.

a. local maximum
b. local minimum
c. neither
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Give the number of intervals where the graph of $f$ is concave up.

a. 1  

b. 2  

c. 3  

d. 4  

e. None of these
Assume the domain of \( f \) is all real numbers. The graph of \( f'(x) \) is shown below. Give the number of intervals where the graph of \( f \) is concave down.

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) None of these
Assume the domain of $f$ is all real numbers. The graph of $f'(x)$ is shown below. Give the number of the points of inflection of the graph of $f$.

a. 1  

b. 2  

c. 3  

d. 4  

e. None of these
The line $x = c$ is a **vertical asymptote** for the function $f$:

$$f(x) \to \infty \quad \text{as} \quad x \to c.$$
The line $x = c$ is a **vertical asymptote** for the function $f$:

$$f(x) \to -\infty \quad \text{as} \quad x \to c.$$
The line $x = c$ is a vertical asymptote for both functions $f$ and $g$:

$$f(x) \to \infty \text{ and } g(x) \to -\infty \quad \text{as } x \to c^-.$$
The line \( x = c \) is a **vertical asymptote** for both functions \( f \) and \( g \):

\[
f(x) \to \infty \quad \text{and} \quad g(x) \to -\infty \quad \text{as} \quad x \to c^+.
\]
Typically, to locate the vertical asymptotes for a function $f$,

- find the values $x = c$ at which $f$ is discontinuous
- and determine the behavior of $f$ as $x$ approaches $c$.

The vertical line $x = c$ is a vertical asymptote for $f$ if any one of the following conditions holds

- $f(x) \to \infty$ or $-\infty$ as $x \to c^+$;
- $f(x) \to \infty$ or $-\infty$ as $x \to c^-$;
- $f(x) \to \infty$ or $-\infty$ as $x \to c$. 
The line $x = 4$ is a **vertical asymptote** for

$$f(x) = \frac{3x + 6}{x^2 - 2x - 8} = \frac{3(x + 2)}{(x + 2)(x - 4)}.$$
Vertical Asymptotes: Tangent Function

The line \( x = \pm \pi/2, \pm 3\pi/2, \pm 5\pi/2, \cdots \), are vertical asymptotes for the tangent function.
The line $y = L$ is a **horizontal asymptote** for the function $f$:

$$f(x) \to L \quad \text{as} \quad x \to \infty.$$
The line $y = L$ is a **horizontal asymptote** for the function $f$:

$$f(x) \to L \quad \text{as} \quad x \to -\infty.$$
The line $x = 2$ is a **vertical asymptote**.

The line $y = 1$ is a **horizontal asymptote**.
Let
\[ R(x) = \frac{a_n x^n + \cdots + a_1 x + a_0}{b_k x^k + \cdots + b_1 x + b_0} \]
be a rational function. Then

- if \( n < k \),
  \[ R(x) \rightarrow 0 \quad \text{as} \quad x \rightarrow \pm \infty; \]

- if \( n = k \),
  \[ R(x) \rightarrow \frac{a_n}{b_n} \quad \text{as} \quad x \rightarrow \pm \infty; \]

- if \( n > k \),
  \[ R(x) \rightarrow \pm \infty \quad \text{as} \quad x \rightarrow \pm \infty. \]
The lines $x = \pm 1$ are vertical asymptotes.
The line $y = 3$ is a horizontal asymptote.
The graph of $f(x) = x^{1/3}$ has a vertical tangent at the point $(0,0)$ since

$$f'(x) = \frac{1}{3}x^{-2/3} \to \infty \quad \text{as} \ x \to 0.$$
The graph of $g(x) = (2 - x)^{1/5}$ has a **vertical tangent** at the point $(2, 0)$ since

$$g'(x) = -\frac{1}{5}(2-x)x^{-4/5} \to -\infty \quad \text{as } x \to 2.$$
The graph of $f(x) = x^{2/3}$ has a **vertical cusp** at the point $(0, 0)$ since $f'(x) = \frac{2}{3}x^{-1/3}$ and

$$f'(x) \to -\infty \text{ as } x \to 0^-, \quad \text{and} \quad f'(x) \to \infty \text{ as } x \to 0^+.$$
Vertical Cusp: Rational Power $g(x) = 2 - (x - 1)^{2/5}$

The graph of $g(x) = 2 - (x - 1)^{2/5}$ has a vertical cusp at the point $(1, 2)$ since $g'(x) = -\frac{2}{5}(x - 1)^{-3/5}$ and

$$g'(x) \to \infty \text{ as } x \to 0^-, \quad \text{and} \quad g'(x) \to -\infty \text{ as } x \to 0^+.$$
Example: $f(x) = |x^3 - 1|$

Is there a vertical cusp for the graph of $f(x) = |x^3 - 1|$?
Example

- Give the equations of the vertical asymptotes, if any.
- Give the equations of the horizontal asymptotes, if any.
Example

- Give the equations of the vertical asymptotes, if any.
- Give the equations of the horizontal asymptotes, if any.
- Give the number $c$, if any, at which the graph has a vertical cusp.