Lecture 18
Section 5.5 Some Area Problems

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Quiz 1

What is today?

a. Monday
b. Wednesday
c. Friday
d. None of these
Area below the graph of a Nonnegative $f$

$f(x) \geq 0$ for all $x$ in $[a, b]$.

$\Omega =$ region below the graph of $f$.

Area of $\Omega = \int_{a}^{b} f(x) \, dx = F(b) - F(a)$

where $F(x)$ is an antiderivative of $f(x)$.
Fundamental Theorem of Integral Calculus

Theorem

In general,
\[ \int_a^b f(x) \, dx = F(b) - F(a). \]

where \( F(x) \) is an antiderivative of \( f(x) \).

<table>
<thead>
<tr>
<th>Function</th>
<th>Antiderivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^r )</td>
<td>( \frac{x^{r+1}}{r+1} ) ( (r ) a rational number ( \neq -1 )</td>
</tr>
<tr>
<td>( \sin x )</td>
<td>( -\cos x )</td>
</tr>
<tr>
<td>( \cos x )</td>
<td>( \sin x )</td>
</tr>
<tr>
<td>( \sec^2 x )</td>
<td>( \tan x )</td>
</tr>
<tr>
<td>( \sec x \tan x )</td>
<td>( \sec x )</td>
</tr>
<tr>
<td>( \csc^2 x )</td>
<td>( -\cot x )</td>
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<tr>
<td>( \csc x \cot x )</td>
<td>( -\csc x )</td>
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</tbody>
</table>
Quiz 2

Give the value of \( \int_{-1}^{1} [x^3 - 2x^2 + \sin(\pi x)] \, dx \).

a. \( \frac{1}{2} \)

b. \( \frac{4}{3} \)

c. \( -\frac{4}{3} \)

d. \( -\frac{1}{2} \)

e. None of these
Example 1

Find the area below the graph of the square-root function from \( x = 0 \) to \( x = 1 \).
Example 2

Example

Find the area bounded above by the curve $y = 4 - x^2$ and below by the $x$-axis.
Give the area bounded between the x-axis and the graph of $y = x^2 + 1$ for $-1 \leq x \leq 2$.

a. 5

b. 4

c. 3

d. 2

e. None of these
Area between the graphs of two Nonnegative $f$ and $g$

$f(x) \geq g(x) \geq 0$ for all $x$ in $[a, b]$.

$\Omega =$ region between the graphs of $f$ (Top) and $g$ (Bottom).

Area of $\Omega = \int_a^b \left[ \text{Top} - \text{Bottom} \right] \, dx = \int_a^b \left[ f(x) - g(x) \right] \, dx$. 
Example 3

Find the area bounded above by $y = x + 2$ and below by $y = x^2$. 
Area between the graphs of $f$ and $g$

\[ f(x) \geq g(x) \quad \text{for all } x \text{ in } [a, b]. \]

\[ \Omega = \text{region between the graphs of } f \text{ (Top)} \text{ and } g \text{ (Bottom)}. \]

\[ \text{Area of } \Omega = \int_a^b \left[ \text{Top} - \text{Bottom} \right] \, dx = \int_a^b \left[ f(x) - g(x) \right] \, dx. \]
Example 4

Find the area of the region shown in the figure below.
Example 5

Find the area between \( y = 4x \) and \( y = x^3 \) from \( x = -2 \) to \( x = 2 \).
Example 6

Example

Use integrals to represent the area of the region $\Omega = \Omega_1 \cup \Omega_2$ shaded in the figure below.
\( \int_a^c f(x) \, dx \) as Signed Area

\[ f(x) \geq 0 \quad \text{for all } x \text{ in } [a, b] \]

\[ \int_a^b f(x) \, dx = \text{Area of } \Omega_1 \]

\[ f(x) \leq 0 \quad \text{for all } x \text{ in } [b, c] \]

\[ \int_b^c f(x) \, dx = -\text{Area of } \Omega_2 \]

\[ \int_a^c f(x) \, dx = \int_a^b f(x) \, dx + \int_b^c f(x) \, dx = \text{Area of } \Omega_1 - \text{Area of } \Omega_2 \]

\[ = \text{Area above the } x\text{-axis} - \text{Area below the } x\text{-axis} \]
\[ \int_{a}^{b} f(x) \, dx = \int_{a}^{c} f(x) \, dx + \int_{c}^{d} f(x) \, dx + \int_{d}^{e} f(x) \, dx + \int_{e}^{b} f(x) \, dx \]

\[ = \text{Area of } \Omega_1 - \text{Area of } \Omega_2 + \text{Area of } \Omega_3 - \text{Area of } \Omega_4 \]

\[ = [\text{Area of } \Omega_1 + \text{Area of } \Omega_3] - [\text{Area of } \Omega_2 + \text{Area of } \Omega_4] \]

\[ = \text{Area above the } x\text{-axis} - \text{Area below the } x\text{-axis}. \]
Example 7

Evaluate $\int_{-1}^{3} (x^2 - 2x) \, dx$ and interpret the result in terms of areas.
Example 8

Example

Use integrals to represent the area of the region shaded in the figure below.
The graph of $y = f(x)$ is shown below. $\Omega_1$ has area $\frac{4}{3}$, $\Omega_2$ has area $\frac{4}{3}$, and $\Omega_3$ has area $\frac{4}{3}$. Give $\int_{-1}^{3} f(x) \, dx$.

a. 0  
b. $\frac{4}{3}$  
c. $\frac{8}{3}$  
d. 4  
e. None of these
The graph of $y = f(x)$ is shown below. $\Omega_1$ has area $\frac{4}{3}$, $\Omega_2$ has area $\frac{4}{3}$, and $\Omega_3$ has area $\frac{4}{3}$. Give $\int_{-1}^{2} f(x) \, dx$.

a. 0  

b. $\frac{4}{3}$  

c. $\frac{8}{3}$  

D. 4  

E. None of these
The graph of \( y = f(x) \) is shown below. \( \Omega_1 \) has area \( \frac{4}{3} \), \( \Omega_2 \) has area \( \frac{4}{3} \), and \( \Omega_3 \) has area \( \frac{4}{3} \). Give \( \int_0^2 f(x) \, dx \).

a. 0

b. \( \frac{4}{3} \)

c. \( \frac{8}{3} \)

d. 4

e. None of these
Quiz 7

The graph of $y = f(x)$ is shown below. $\Omega_1$ has area $\frac{4}{3}$, $\Omega_2$ has area $\frac{4}{3}$, and $\Omega_3$ has area $\frac{4}{3}$. Give $\int_{2}^{3} f(x) \, dx$.

a. 0  

b. $\frac{4}{3}$  

c. 8  

d. 4  

e. None of these
The graph of \( y = f(x) \) is shown below. \( \Omega_1 \) has area \( \frac{4}{3} \), \( \Omega_2 \) has area \( \frac{4}{3} \), and \( \Omega_3 \) has area \( \frac{4}{3} \). Give \( \int_{0}^{3} f(x) \, dx \).

a. 0  
b. \( \frac{4}{3} \)  
c. 8  
d. 4  
e. None of these
Quiz 9

The graph of $y = f(x)$ is shown below. $\Omega_1$ has area $\frac{4}{3}$, $\Omega_2$ has area $\frac{4}{3}$, and $\Omega_3$ has area $\frac{4}{3}$. Give the area bounded between the x-axis and $y = f(x)$ from $x = -1$ to $x = 3$.

a. 0  

b. $\frac{4}{3}$  

c. $\frac{8}{3}$  

d. 4  

e. None of these