Sample Problems, Exam 2

Part I. Techniques of Integration

1. \[ \int \frac{1}{x^2 \sqrt{1 - x^2}} \, dx \] \hspace{1cm} \text{Answer:} \\

2. \[ \int \frac{1}{x^4 - 16} \, dx \] \hspace{1cm} \text{Answer:} \\

3. \[ \int \frac{x^2 + 8x - 3}{x^3 + 3x^2} \, dx \] \hspace{1cm} \text{Answer:} \\

4. \[ \int \frac{x}{\sqrt{4 + x^2}} \, dx \] \hspace{1cm} \text{Answer:} \\

5. \[ \int_0^1 \frac{x^2}{(4 - x^2)^{3/2}} \, dx \] \hspace{1cm} \text{Answer:} \\

6. \[ \int \frac{x^2 + 2x - 4}{x^3 - 4x} \, dx \] \hspace{1cm} \text{Answer:} \\

Part II. Numerical Integration

Set \( f(x) = x^2 + 1 \) on \([0, 4]\).

1. Use the midpoint rule with \( n = 4 \) to approximate \( \int_0^4 f(x) \, dx \).

Answer: \\

2. Use the trapezoidal rule with \( n = 4 \) to approximate \( \int_0^4 f(x) \, dx \).

Answer: \\

3. Use Simpson’s rule with \( n = 2 \) to approximate \( \int_0^4 f(x) \, dx \).

Answer: \\

4. Determine the smallest integer \( n \) such that the trapezoidal approximation \( T_n \) approximates \( \int_0^4 f(x) \, dx \) with error less than 0.005.

Answer: 

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Part III. Polar Coordinates

1. Give the rectangular coordinates of the point with polar coordinates \([-2, 8\pi/3]\).

Answer

2. Give all possible polar coordinates for the point with rectangular coordinates \((-4\sqrt{3}, 4)\).

Answer

3. Sketch the graph of \(r = 1 + 2\cos \theta, \ 0 \leq \theta \leq 4\pi/3\).

Answer

4. The graphs of \(C_1: r = 2 - \cos \theta\) and \(C_2: r = 1 + \cos \theta\) are shown in the figure. Calculate the area of the region inside \(C_2\) and outside \(C_1\).

Answer

5. Find the polar equation for \((x^2 + y^2)^2 = 4xy\).

Answer

6. Write the equation \(r = 4\sin \theta\) in rectangular coordinates.

Answer

Parametric Equations

1. Express the curve \(x = 2 + \sin t, \ y = -1 + \cos t\) by an equation in \(x\) and \(y\).

Answer
2. Find a parametrization of the line segment from \((-2, 3)\) to \((1, 5)\).

Answer

3. Find a parametrization for the curve \(y^3 = x^2\) from \((1, 1)\) to \((8, 4)\).

Answer

4. Give an equation for the normal line to the graph of \(x = \sin t, y = 2 + \cos 2t\) at the point where \(t = \pi/6\).

Answer

5. Give an equation for the line tangent to the polar curve \(r = 2 \cos \theta\) at the point where \(\theta = \pi/3\).

Answer

6. Find the points \((x, y)\) at which the curve \(x = t^2 - 2t, y = t^3 - 12t\) has (a) a horizontal tangent, (b) a vertical tangent.

Answer: 

7. A particle moves along the curve \(x = 1 + \frac{1}{2}t^2, y = 2 + \frac{2}{3}t^3, 0 \leq t \leq 2\). (a) What is the speed of the particle at time \(t\)? (b) What is the total distance traveled by the particle?

Answer:

8. Find the length of the polar curve \(r = 1 - \cos \theta\), \(0 \leq \theta \leq 2\pi\).

Answer

Part IV. Sequences

1. Determine a formula for \(a_n\), the general term of the given sequence. Then determine whether the sequence converges and if it does, give the limit.

\[(i)\] \(4, 1, \frac{1}{4}, \frac{1}{16}, \ldots\) \[(ii)\] \(
\frac{2}{1}, \left(\frac{3}{2}\right)^2, \left(\frac{4}{3}\right)^3, \left(\frac{5}{4}\right)^4, \ldots
\)

Answer:
2. Determine whether or not the given sequence is bounded above, bounded below, bounded. If it is bounded above or below, give the least upper and/or greatest lower bounds.

(i) \( \{ \cos(n\pi/3) \} \)  
(ii) \( \left\{ \frac{n^3 + 1}{n^2 + 2n + 3} \right\} \)  
(iii) \( \left\{ 2 + \frac{(-1)^n}{n} \right\} \)

Answer: 

3. Determine the monotonicity of the given sequence.

(i) \( \{(2/3)^n\} \)  
(ii) \( \left\{ \frac{n^2}{n + 2} \right\} \)  
(iii) \( \left\{ \frac{n + (-1)^n}{n^2} \right\} \)

Answer: 

4. Determine whether or not the given sequence converges or diverges. If it converges, give the limit.

(i) \( \left\{ \frac{n^2 + 1}{\sqrt{4n^4 + 2n^2 + 1}} \right\} \)  
(ii) \( \left\{ \frac{\sin^2 n}{n} \right\} \)  
(iii) \( \left\{ \frac{(-1)^n(2n)}{\sqrt{n^2 + 4}} \right\} \)

Answer: 

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