Lecture 5
Section 7.6 Exponential Growth and Decay

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Annual population growth rate is the increase in a country’s population during one year, expressed as a percentage of the population at the start of that period.
Let $P(t)$ the size of the population $P$ at time $t$. Then the growth rate

$$k = \frac{P(t + 1) - P(t)}{P(t)} \approx \frac{1}{P(t)} \lim_{h \to 0} \frac{P(t + h) - P(t)}{h} = \frac{P'(t)}{P(t)}.$$
Countries with the most rapid population growth rates tend to be located in Africa and the Middle East.
Countries with the **slowest** population growth rates tend to be located in Europe and North America.
**Theorem**

If

\[ f'(t) = kf(t) \]

for all \( t \) in some interval

then \( f \) is an exponential function

\[ f(t) = Ce^{kt} \]

where \( C \) is arbitrary constant. If the initial value of \( f \) at \( t = 0 \) is known, then

\[ C = f(0), \quad f(t) = f(0)e^{kt} \]

**Proof.**

\[
\frac{f'(t)}{f(t)} = k \quad \Rightarrow \quad \frac{d}{dt} \ln f(t) = k \\
\ln f(t) = kt + c \quad \Rightarrow \quad f(t) = e^{kt+c} = Ce^{kt} \\
f(0) = Ce^0 = C \quad \Rightarrow \quad f(t) = f(0)e^{kt}
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The population was 4.5 billion in 1980, and 6 billion in 2000.

Let $P(t)$ be the population (in billion) $t$ years after 1980 and $k$ the annual population growth rate.

$P'(t) = kP(t)$ with $P(0) = 4.5$ gives $P(t) = 4.5e^{kt}$.

$P(20) = 6 \Rightarrow 4.5e^{k20} = 6$, $20k = \ln(6/4.5)$, $k \approx 1.43\%$. 
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  - \( P(20) = 6 \) \( \Rightarrow \) \( 4.5e^{k20} = 6 \), \( 20k = \ln(6/4.5) \), \( k \approx 1.43\% \).
When Will the World Population Double?

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When Will the World Population Double?

How long will it take for the population to double from 1980?

To find the “double time”, we solve $2P(0) = P(0)e^{kt}$ for $t$.

1. $e^{kt} = 2$, $kt = \ln 2$, $t = \frac{\ln 2}{k} \approx \frac{0.69}{k\%} = \frac{69}{1.43} \approx 48.5$

2. The population will double in 48.5 years (from 1980); that is, the population will reach 9 billion midyear in the year 2028.
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Future population projections are notoriously inaccurate:
\[ \frac{d}{dk} \left( P(48.5) \right) = P(0) \frac{d}{dk} \left( e^{48.5k} \right) = 48.5 \ P(48.5). \]

A difference of just 0.1% between predicted and actual growth rates translates into hundreds of millions of lives:
\[ 48.5 \times 9 \times 0.1\% \approx 0.43 \text{ billion}. \]
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A sustainable growth of the world population **cannot be an exponential growth**. But what is it?!
### Quiz

1. \( \lim_{x \to -\infty} e^x =: \) (a) 0, (b) \(-\infty\), (c) \(\infty\).

2. \( \lim_{x \to \infty} e^x =: \) (a) 0, (b) \(-\infty\), (c) \(\infty\).
Radioactive Waste

- Radioactive waste (or nuclear waste) is a material deemed no longer useful that has been contaminated by or contains radionuclides.
- Radionuclides are unstable atoms of an element that decay, or disintegrate spontaneously, emitting energy in the form of radiation.
- Radioactive waste has been created by humans as a by-product of various endeavors since the discovery of radioactivity in 1896 by Antoine Henri Becquerel.

Workers at a nuclear power plant standing near a storage pond filled with spent fuel.
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A half-life is a measure of time required for an amount of radioactive material to decrease by one-half of its initial amount. The half-life of a radionuclide can vary from fractions of a second to millions of years: sodium-26 (1.07 seconds), hydrogen-3 (12.3 years), carbon-14 (5,730 years), uranium-238 (4.47 billion years).
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Let $A(t)$ be the amount of a radioactive material present at time $t$ and $k < 0$ the decay constant. Then

$$A'(t) = kA(t)$$

gives:

$$A(t) = A(0)e^{kt}.$$ 

Let $T_{\text{half}}$ denote the half-life. Then

$$\frac{1}{2}A(0) = A(0)e^{kT_{\text{half}}},$$

$$T_{\text{half}} = \frac{-\ln 2}{k} \approx \frac{-0.69}{k}.$$
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Cobalt-60

- Cobalt-60, used extensively in medical radiology, has a half-life of 5.3 years. The decay constant $k$ is given by

$$k = \frac{-\ln 2}{T_{\text{half}}} \approx \frac{-0.69}{5.3} \approx -0.131.$$  

- If the initial sample of cobalt-60 has a mass of 100 grams, then the amount of the sample that will remain $t$ years after is

$$A(t) = A(0)e^{kt} = 100e^{-0.131t}.$$
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The mathematics of radioactive decay is useful for many branches of science far removed from nuclear physics.

One reason is that, in the late 1940’s, Willard F. Libby discovered natural carbon-14 (radiocarbon), a radioactive isotope of carbon with a half-life of 5730 years.

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All living organisms take in carbon through their food supply. While living, the ratio of radiocarbon to nonradioactive carbon that makes up the organism stays constant, since the organism takes in a constant supply of both in its food. After it dies, however, it no longer takes in either form of carbon.

The ratio of radiocarbon to nonradioactive carbon then decreases with time as the radiocarbon decays away. The ratio decreases exponentially with time, so a 5600-year-old organic object has about half the radiocarbon/carbon ratio as a living organic object of the same type today.
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Quiz (cont.)

3. \( \frac{d}{dx} e^{-x} =: \) (a) \(-e^{-x}\), (b) \(e^x\), (c) \(e^{-x}\).

4. \( \frac{d^2}{dx^2} e^{-x} =: \) (a) \(-e^{-x}\), (b) \(e^x\), (c) \(e^{-x}\).
Compounding

The value, at the end of the year, of a principle of $1000 invested at 6% compounded:

- annually (once per year):
  \[ A(1) = 1000(1 + 0.06) = $1060. \]

- quarterly (4 times per year):
  \[ A(1) = 1000\left(1 + \left(\frac{0.06}{4}\right)\right)^4 \approx $1061.36. \]

- monthly (12 times per year):
  \[ A(1) = 1000\left(1 + \left(\frac{0.06}{12}\right)\right)^{12} \approx $1061.67. \]

- continuously:
  \[ A(1) = 1000 \lim_{n \to \infty} \left(1 + \frac{0.06}{n}\right)^n. \]

Let \( x = \frac{n}{0.06}. \)

\[ A(1) = 1000 \left[ \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x \right]^{0.06} \]
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**Theorem**

\[ e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x \]

**Proof.**

- Define \( g(h) = \begin{cases} \frac{1}{h} \ln(1 + h), & h \in (-1, 0) \cup (0, \infty), \\ 1, & h = 0 \end{cases} \)
- At \( x = 1 \), the logarithm function has derivative
  \[ (\ln x)'\big|_{x=1} = \lim_{h \to 0} \frac{\ln(1 + h) - \ln 1}{h} = \frac{1}{x}\bigg|_{x=1} \]
  thus \( g \) is continuous at 0.
- The composition \( e^g(h) \) is continuous at 0;
  \[ \lim_{h \to 0} (1 + h)^{\frac{1}{h}} = e. \]
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The economists’ formula for continuous compounding is

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Outline

- Population Growth
  - Human Population Growth

- Radioactive Decay
  - Radioactive Decay

- Compound Interest
  - Compound Interest
Online Resources

- www.unesco.org/education
- www.populationinstitute.org
- www.globalchange.umich.edu
- www.geohive.com
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