

Math 2331 – Linear Algebra

2.3 Characterizations of Invertible Matrices

Key Exercises 15-24

Jiwen He

Department of Mathematics, University of Houston

`jiwenhe@math.uh.edu`
`math.uh.edu/~jiwenhe/math2331`



2.3 Characterizations of Invertible Matrices

Key Exercises 15–24

- The Invertible Matrix Theorem ties together most of the concepts studied thus far.
- Additional statements are added to the theorem in Sections 2.9, 3.2, 4.6, 5.2 and 7.4.
- Key Exercises: 15–24



15. Is it possible for a 4×4 matrix to be invertible when its columns do not span \mathbb{R}^4 ? Why or why not?



- 16.** If an $n \times n$ matrix A is invertible, then the columns of A^T are linearly independent. Explain why.



17. Can a square matrix with two identical columns be invertible? Why or why not?



- 18.** Can a square matrix with two identical rows be invertible? Why or why not?



- 19.** If the columns of a 7×7 matrix D are linearly independent, what can be said about the solutions of $D\mathbf{x} = \mathbf{b}$? Why?



- 20.** If A is a 5×5 matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^5 , is it possible that for some \mathbf{b} , the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution? Why or why not?



21. If the equation $C\mathbf{u} = \mathbf{v}$ has more than one solution for some \mathbf{v} in \mathbb{R}^n , can the columns of the $n \times n$ matrix C span \mathbb{R}^n ? Why or why not?



- 22.** If $n \times n$ matrices E and F have the property that $EF = I$, then E and F commute. Explain why.



23. Assume that F is an $n \times n$ matrix. If the equation $F\mathbf{x} = \mathbf{y}$ is inconsistent for some \mathbf{y} in \mathbb{R}^n , what can you say about the equation $F\mathbf{x} = \mathbf{0}$? Why?



24. If an $n \times n$ matrix G cannot be row reduced to I_n , what can you say about the columns of G ? Why?

