Math 3331  Differential Equations
2.5 Mixing Problems

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2.5 Mixing Problems

- Balance Law
- Mixture of Water and Salt
  - Example 5.1
  - Example 5.3
Solution of a mixture of water and salt

\( x(t) \): amount of salt  
\( V(t) \): volume of the solution  
\( c(t) \): concentration of salt

\[ c(t) = \frac{x(t)}{V(t)} \]

Balance Law

\[ \frac{d}{dt} \left( x(t) \right) = \text{rate in} - \text{rate out} \]

rate = flow rate \( \times \) concentration
Example 1

See Text, Example 2.5.1

\( V(t) = 100 \text{ gal}, \text{ kept constant} \)

concentration in = 2 lb/gal

flow rate in = 3 gal/min

flow rate out = flow rate in

\[ \Rightarrow c(t) = \frac{x(t)}{100} \text{ lb/gal} \]

\[ \Rightarrow \text{ rate in } = 6 \text{ lb/min} \]

\[ \Rightarrow \text{ rate out } = 3x(t)/100 \text{ lb/min} \]

Balance Law

\[ \frac{dx}{dt} = 6 - 3x/100 \]
Example 2

See Text, Example 2.5.2

concentration in = 1.5 lb/gal
flow rate in = 3 gal/min
flow rate out = 1 gal/min

\[ V(0) = 300 \text{ gal} \]

\[ \Rightarrow V(t) = 300 + 2t \text{ lb/gal} \]

\[ \Rightarrow c(t) = \frac{x(t)}{(300 + 2t)} \text{ lb/gal} \]

\[ \Rightarrow \text{rate in } = 4.5 \text{ lb/min} \]

\[ \Rightarrow \text{rate out } = \frac{x(t)}{(300 + 2t)} \text{ lb/min} \]

Balance Law

\[ \frac{dx}{dt} = 4.5 - \frac{x}{(300 + 2t)} \]