Probability

Second Exam **MATH 3338-10853 (Fall 2006)** **September 25, 2006**

This exam has 3 questions, for a total of 100 points. Please answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name and UH-ID: ______________________________

1. A box in a certain supply room contains four 40-W lightbulbs, five 60-W bulbs, and six 75-W bulbs. Suppose that three bulbs are randomly selected.

   (a) What is the probability that exactly two of the selected bulbs are rated 75 W?

   (b) What is the probability that all three of the selected bulbs have the same rating?

   Solution:

   (a)
$$P(\text{all three are 75-W}) = \frac{C_{3,6}}{C_{3,15}} = \frac{20}{455} \approx 0.0440$$

   (b)
$$P(\text{all three are the same}) = \frac{C_{3,4} + C_{3,5} + C_{3,6}}{C_{3,15}} = \frac{4 + 10 + 20}{455} \approx 0.0747$$

2. A mathematics department has tried a different text for statistics during each of the last three quarters. During the fall quarter, 50 students used the text by Professor 1; during the winter quarter, 30 students used the text by Professor 2; and during the spring quarter, 20 students used the text by Professor 3. A survey at the end of each quarter showed that 20 students were satisfied with Professor 1's book, 15 were satisfied with Professor 2's book, and 16 were satisfied with Professor 3's book. If a student who took statistics during one of these quarters is selected at random and admits to having been satisfied with the text, is the student most likely to have used the book by Professor 1, Professor 2, or Professor 3? Please use a tree diagram for your answer.

   Solution: Let $\Omega = \{\text{students taking statistics}\}$, $A_1 = \{\text{students in Professor 1's class}\}$, $A_2 = \{\text{students in Professor 2's class}\}$, $A_3 = \{\text{students in Professor 3's class}\}$, and $B = \{\text{students are satisfied}\}$. Then, $|A_1| = 50$, $|A_2| = 30$, $|A_3| = 20$, and $|\Omega| = 50+30+20 = 100$. We have

$$P(A_1) = \frac{|A_1|}{|\Omega|} = \frac{50}{100} = 0.5, \quad P(A_2) = \frac{|A_2|}{|\Omega|} = \frac{30}{100} = 0.3, \quad P(A_3) = \frac{|A_3|}{|\Omega|} = \frac{20}{100} = 0.2,$$

   and

$$\begin{aligned} P(B|A_1) &= \frac{|A_1 \cap B|}{|A_1|} = \frac{20}{50} = 0.4, \\ P(B|A_2) &= \frac{|A_2 \cap B|}{|A_2|} = \frac{15}{30} = 0.5, \\ P(B|A_3) &= \frac{|A_3 \cap B|}{|A_3|} = \frac{16}{20} = 0.8. \end{aligned}$$

   Then

$$\begin{aligned} P(A_1 \cap B) &= P(A_1)P(B|A_1) = 0.5 \times 0.4 = 0.20 \ \left(= \frac{|A_1 \cap B|}{|\Omega|}\right) \\ P(A_2 \cap B) &= P(A_2)P(B|A_2) = 0.3 \times 0.5 = 0.15 \ \left(= \frac{|A_2 \cap B|}{|\Omega|}\right) \\ P(A_3 \cap B) &= P(A_3)P(B|A_3) = 0.2 \times 0.8 = 0.16 \ \left(= \frac{|A_3 \cap B|}{|\Omega|}\right) \end{aligned}$$

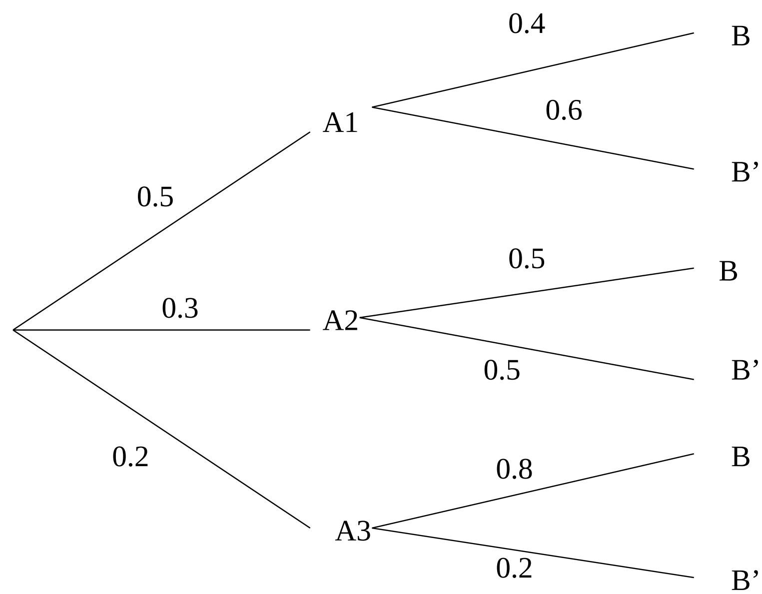
   We have

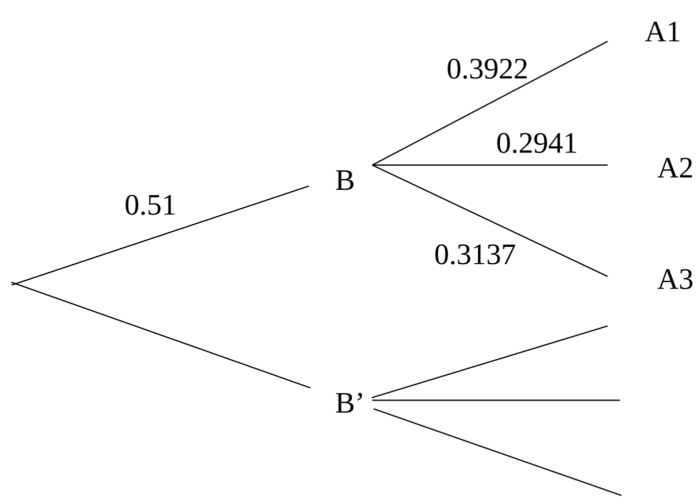
$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = 0.20 + 0.15 + 0.16 = 0.51 \ \left(= \frac{|B|}{|\Omega|}\right)$$

   and

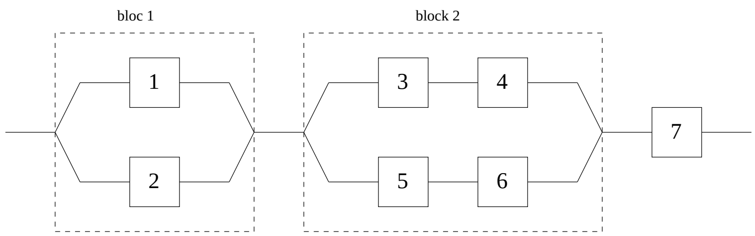
$$\begin{aligned} P(A_1|B) &= \frac{P(A_1 \cap B)}{P(B)} = \frac{0.20}{0.51} \approx 0.3922 \\ P(A_2|B) &= \frac{P(A_2 \cap B)}{P(B)} = \frac{0.15}{0.51} \approx 0.2941 \\ P(A_3|B) &= \frac{P(A_3 \cap B)}{P(B)} = \frac{0.16}{0.51} \approx 0.3137 \end{aligned}$$

   A student satisfied with the text is most likely to have used the book by Professor 1


0.5
0.3
0.2
A1
A2
A3
0.4
0.6
0.5
0.5
0.8
0.2
B
B'
B
B'
B
B'


0.51
B
B'
0.3922
0.2941
0.3137
A1
A2
A3

3. Consider the system of components connected as in the accompanying picture.



For a subsystem connected in parallel to function, at least one of the components or subsystems in the parallel must work. For a subsystem connected in series to function, all the components or subsystems in the series must work. If the components work independently of one another and $p_i = P(\text{component } i \text{ works}) = 0.9$ for $i = 1, \ldots, 7$, calculate $P(\text{system works})$.

Solution: First, the probability of block 1 in parallel working is

$$P(\text{block 1 working}) = 1 - (1 - p_1)(1 - p_2) = 1 - (1 - 0.9)^2 = 0.99$$

and the probability of block 2 in seris-parallel is

$$P(\text{block 1 working}) = 1 - (1 - p_3p_4)(1 - p_5p_6) = 1 - (1 - 0.9^2)^2 = 0.9639$$

The system is in series of block 1, block2, and component 7, then the probability of the system working is

$$P(\text{system working}) = 0.99 \times 0.9639 \times 0.9 \approx 0.8588$$

When you finish this exam, you should go back and reexamine your work for any errors that you may have made.