HW14 Solutions
Math 3388-10853: Probability (Fall 2006), Dr. Jiwen He

1.

| P(x_1) | .20 | .50 | .30 |
| P(x_2) | x_2 | x_1 |
| 25 | 40 | 65 |
| 20 | 25 | 04 | 10 | 06 |
| 50 | 40 | 10 | 25 | 15 |
| 30 | 65 | 06 | 15 | 09 |

a. 

| \bar{x} | 25 | 32.5 | 40 | 45 | 52.5 | 65 |
| p(\bar{x}) | .04 | .20 | .25 | .12 | .30 | .09 |

\[ E(\bar{x}) = 25(.04) + 32.5(.20) + \ldots + 65(.09) = 44.5 = \mu \]

b. 

| s^2 | 0 | 112.5 | 312.5 | 800 |
| P(s^2) | .38 | .20 | .30 | .12 |

\[ E(s^2) = 212.25 = \sigma^2 \]
4. a. Possible values of M are: 0, 5, 10. M = 0 when all 3 envelopes contain 0 money, hence 
p(M = 0) = ( .5)^3 = .125. M = 10 when there is a single envelope with $10, hence 
p(M = 10) = 1 - p(\text{no envelopes with $10}) = 1 - ( .8)^3 = .488.
p(M = 5) = 1 - [.125 + .488] = .387.

<table>
<thead>
<tr>
<th>M</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(M)</td>
<td>.125</td>
<td>.387</td>
<td>.488</td>
</tr>
</tbody>
</table>

An alternative solution would be to list all 27 possible combinations using a tree diagram and computing probabilities directly from the tree.

b. The statistic of interest is M, the maximum of x₁, x₂, or x₃, so that M = 0, 5, or 10. The population distribution is as follows:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>1/2</td>
<td>3/10</td>
<td>1/5</td>
</tr>
</tbody>
</table>

Write a computer program to generate the digits 0 – 9 from a uniform distribution. Assign a value of 0 to the digits 0 – 4, a value of 5 to digits 5 – 7, and a value of 10 to digits 8 and 9. Generate samples of increasing sizes, keeping the number of replications constant and compute M from each sample. As n, the sample size, increases, p(M = 0) goes to zero, p(M = 10) goes to one. Furthermore, p(M = 5) goes to zero, but at a slower rate than p(M = 0).

11. \( \mu = 12 \text{ cm} \quad \sigma = .04 \text{ cm} \)

a. \( n = 16 \)
\[
E(\bar{X}) = \mu = 12 \text{ cm} \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{.04}{4} = .01 \text{ cm}
\]

b. \( n = 64 \)
\[
E(\bar{X}) = \mu = 12 \text{ cm} \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{.04}{8} = .005 \text{ cm}
\]

c. \( \bar{X} \) is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of \( \bar{X} \) with a larger sample size.

13. a. \( \mu = 50, \sigma_{\bar{X}} = \frac{1}{\sqrt{100}} = .10 \)
\[
P(49.75 \leq \bar{X} \leq 50.25) = P\left(\frac{49.75 - 50}{.10} \leq Z \leq \frac{50.25 - 50}{.10}\right)
= P(-.25 \leq Z \leq .25) = .9876
\]

b. \( P(49.75 \leq \bar{X} \leq 50.25) \approx P\left(\frac{49.75 - 49.8}{.10} \leq Z \leq \frac{50.25 - 49.8}{.10}\right)
= P(-.5 \leq Z \leq 4.5) = .6915
\]
17. \[ X \sim N(10,1), \ n = 4 \]
\[ \mu_{x_0} = n\mu = (4)(10) = 40 \quad \text{and} \quad \sigma_{x_0} = \sigma\sqrt{n} = (2)(1) = 2, \]
We desire the 95\(^{th}\) percentile: \[ 40 + (1.645)(2) = 43.29 \]

21. 
\[ \text{a. With } Y = \# \text{ of tickets, } Y \text{ has approximately a normal distribution with } \mu = \lambda = 50, \]
\[ \sigma = \sqrt{\lambda} = 7.071, \text{ so } P(35 \leq Y \leq 70) \approx \Phi\left(\frac{34.5 - 50}{7.071}\right) - \Phi\left(-2.19\right) \]
\[ \leq Z \leq 2.90) = .9838 \]

\[ \text{b. Here } \mu = 250, \ \sigma^2 = 250, \ \sigma = 15.811, \ \text{so } P(225 \leq Y \leq 275) \approx \]
\[ \Phi\left(\frac{224.5 - 250}{15.811}\right) \leq Z \leq \frac{275.5 - 250}{15.811} \right) = \Phi(-1.61) = .8926 \]