Math 3338: Probability (Fall 2006)

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`http://math.uh.edu/~jiwenhe/math3338fall06.html`

2.5 Independence

Definition of independent events

- **Definition:** Two events A and B are *independent* if

$$P(A|B) = P(A)$$

  and are dependent otherwise.
  - The equality in the definition implies the following equality (and vice versa)

$$P(B|A) = P(B)$$

  - It is also straightforward to show that if A and B are independent, then so are the following pairs of events: (1) A' and B, (2) A and B', and (3) A' and B'.
- **2.31: Tossing a die** $A = \{2, 4, 6\}$, $B = \{1, 2, 3\}$, and $C = \{1, 2, 3, 4\}$. We have

$$P(A) = \frac{1}{2}, \quad P(A|B) = \frac{1}{3}, \quad P(A|C) = \frac{1}{2}.$$

  That is, A and B are dependent, whereas A and C are independent.
- **2.32** Let A and B be mutually exclusive with $P(A) > 0$. Since $A \cap B = \emptyset$, then $P(A|B) = 0 \neq P(A)$, so A and B can not be independent. For example, $A = \{carisblue\}$ and $B = \{carisred\}$, A and B are mutually exclusive, then dependent.

$P(A \cap B)$ When A and B are Independent

- **Proposition:** A and B are independent if and only if

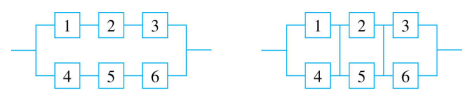
$$P(A \cap B) = P(A) \cdot P(B)$$

- **Proof:** $P(A \cap B) = P(A|B) \cdot P(B) = P(A) \cdot P(B)$ where the second equality is valid if and only if A and B are independent.

- **Definition of the independence of more than two events:** Events $A_1, \ldots, A_n$ are *mutually independent* if for every k $(k = 2, 3, \ldots, n)$ and every subset of indices $i_1, \ldots, i_k$,

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \cdots P(A_{i_k})$$

- **2.35:** Let A_i denote the event that the lifetime of cell i exceeds t_0 $(i = 1, \ldots, 6)$. We assume that A_i's are independent events and that $P(A_i) = .9$ for every i since the cells are identical.



(a) (b)

$$\begin{aligned} P(\text{system lifetime exceeds } t_0) &= P[(A_1 \cap A_2 \cap A_3) \cup (A_4 \cap A_5 \cap A_6)] \\ &= P(A_1 \cap A_2 \cap A_3) + P(A_4 \cap A_5 \cap A_6) \\ &\quad - P[(A_1 \cap A_2 \cap A_3) \cap (A_4 \cap A_5 \cap A_6)] \\ &= (.9)(.9)(.9) + (.9)(.9)(.9) - (.9)(.9)(.9)(.9)(.9)(.9) = .927 \end{aligned}$$