Section 1.6 Problem Notes

1.6.34.

- For part (a), most people showed that W_1 has a basis β, extended it to a basis α for V and defined W_2 to be the subspace with basis $\alpha \setminus \beta$. This is fine, but you still have to show that the two subspaces direct sum to V. Problem 33(b) gives this result, but because this was not assigned you cannot simply assume its result; you have to show it to use in problem 34.
- The most popular choices were $W_2 = \{(0, y) : y \in \mathbb{R}\}$ (which is the y-axis) and $W_2' = \mathrm{Span}\{(1, 1)\}$. I didn't require proof that $W_1 \oplus W_2 = V = W_1 \oplus W_2'$, but make sure you know how to show those two equalities.
- Some people thought that the y-axis was the only complementary subspace. This would be the case if we wanted the subspaces to be orthogonal (something we will get to once we learn about inner products), but all we want is to find another subspace W_2 with $V = W_1 \oplus W_2$. This works if you take any $v \in V \setminus W_1$, so $V = W_1 \oplus \mathrm{Span}\{v\}$.

Section 2.1 Problem Notes

2.1.11.

- Some of you took a complicated route for this problem when you tried to find the image of $(8, 11)$. Here's the easy way:

  Note that $(8, 11) = 2(1, 1) + 3(2, 3)$. Since T is linear

$$T(8, 1) = T(2(1, 1) + 3(2, 3)) = 2T(1, 1) + 3T(2, 3) = 2(1, 0, 2) + 3(1, -1, 4) = (5, -3, 16).$$

Section 2.2 Problem Notes

2.2.16.

- When you are using results from a theorem be sure to quote them more carefully. Instead of saying "Using the basis $\beta = \{v_1, ..., v_n\}$ for V we get a basis for $R(T)$ $\{u_{k+1}, ..., u_n\}$ with $u_{k+i} = T(v_{k+i})$" be more specific: "Following the dimension theorem let $\{v_1, ..., v_k\}$ be a basis for the nullspace of T, then extend this basis to a basis $\beta = \{v_1, ..., v_n\}$ for V. Using the proof of the dimension theorem we have a basis for $R(T)$ given by $\alpha = \{u_{k+1}, ...u_n\}$ by defining $u_{k+i} = T(v_{k+i})$ for $1 \le i \le n - k$. Since the dimensions of V and W are the same, we extend α by k vectors to get a basis $\gamma = \{u_1, ..., u_n\}$ for W.
- For the last step, you should explain why

$$[T]_\beta^\gamma = \begin{pmatrix} 0 & 0 \\ 0 & I_{n-k} \end{pmatrix}$$

  For instance: Since $T(v_i) = 0$ for $i \le k$ and $T(v_i) = u_i$ for $i > k$.