Section 5.1 Problem Notes

5.1.8.

(a) $\det T = 0$ iff. $N(T - \lambda I) = 0$ doesn't make sense when you don't substitute a value for λ. Even then $\det T = 0$ iff. $N(T - 0I) = 0$ is not an obvious fact and requires some explanation.

(b) It is not enough to just say $Tv = \lambda v$ iff. $T^{-1}v = \lambda^{-1}v$. You've got to prove this statement:

$$Tv = \lambda v$$
$$T^{-1}(Tv) = T^{-1}(\lambda v)$$
$$v = \lambda T^{-1}(v)$$
$$\lambda^{-1}v = T^{-1}(v)$$

The above follows because part (a) shows that $\lambda \neq 0$ and T^{-1} is linear so $T^{-1}(\lambda v) = \lambda T^{-1}(v)$.

5.1.12. (b) Different matrix representations for the same linear transformation are similar and thus have the same characteristic polynomial by part (a).

Section 5.2 Problem Notes

5.1.3. (c) It's not enough to calculate the characteristic polynomial and say that it doesn't split. You have to explain why it doesn't split. In this case it factors into $(2 - \lambda)(\lambda^2 + 1)$ and $\lambda^2 + 1$ doesn't have solutions over $\mathbb{R}$.

(e)Keep in mind that the order of the basis is important for diagonalizing. I.e., the eigenvectors/basis elements should be in the same order as the eigenvalues you get in the diagonal. So, the transformation has the diagonal matrix

$$\begin{pmatrix} 1+i & 0 \\ 0 & 1-i \end{pmatrix}$$

in the basis $\beta = \{(1, 1), (-1, 1)\}$.

But the transformation has the diagonal matrix

$$\begin{pmatrix} 1-i & 0 \\ 0 & 1+ii \end{pmatrix}$$

in the basis $\beta = \{(1-, 1), (1, 1)\}$.