

# Math 4377/6308 Advanced Linear Algebra

## 4.3 Properties of Determinants

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## 4.3 Properties of Determinants

- Determinants of Products of Matrices
- Determinant of Inverse of Matrix
- Determinant of Transpose of Matrix
- Cramer's Rule and Solution of Linear System



# Properties of Determinants: Product

## Theorem (4.7)

For  $A, B \in M_{n \times n}(F)$ ,  $\det(AB) = \det(A) \cdot \det(B)$ .



# Properties of Determinants: Inverse

## Corollary

$A \in M_{nn}(F)$  is invertible if and only if  $\det(A) \neq 0$ . If  $A$  is invertible, then  $\det(A^{-1}) = \frac{1}{\det(A)}$ .



# Properties of Determinants: Transpose

## Theorem (4.8)

For  $A \in M_{n \times n}(F)$ ,  $\det(A^t) = \det(A)$ .



# Properties of Determinants: Cramer's Rule

## Theorem (4.9 - Cramer's Rule)

*Let  $Ax = b$  be a system of  $n$  linear equations in  $n$  unknowns. If  $\det(A) \neq 0$ , it has a unique solution  $x = (x_1, \dots, x_n)^t$  with  $x_k = \frac{\det(M_k)}{\det(A)}$ , where  $M_k$  is  $A$  with column  $k$  replaced by  $b$ .*

