Math 4377/6308 Advanced Linear Algebra

5.3 Matrix Limites and Markov Chains

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- Matrix Limits
- Existence of Limits

Matrix Limits

Definition

Let L, A_1, A_2, $\cdots$ be $n \times p$ matrices with complex entries. The sequence A_1, A_2, $\cdots$ is said to *converge to the limit* L if $\lim_{m\to\infty}(A_m)_{ij} = L_{ij}$ for all $1 \leq i \leq n$ and $1 \leq j \leq p$. If L is the limit of the sequence, we write $\lim_{m\to\infty} A_m = L$.

Theorem (5.12)

Let A_1, A_2, $\cdots$ be a sequence of $n \times p$ matrices with complex entries that converges to L. Then for any $P \in M_{r\times n}(C)$ and $Q \in M_{p\times s}(C)$,

$$\lim_{m\to\infty} PA_m = PL \text{ and } \lim_{m\to\infty} A_m Q = LQ.$$

Matrix Limits (cont.)

Corollary

Let $A \in M_{n\times n}(C)$ be such that $\lim_{m\to\infty} A^m = L$. Then for any invertible $Q \in M_{n\times n}(C)$,

$$\lim_{m\to\infty} (QAQ^{-1})^m = QLQ^{-1}.$$

Existence of Limits

Consider the set consisting of the complex number 1 and the interior of the unit disk: $S = \{\lambda \in \mathbb{C} : |\lambda| < 1 \text{ or } \lambda = 1\}$.

Theorem (5.13)

Let A be a square matrix with complex entries. Then $\lim_{m\to\infty} A^m$ exists if and only if both of the following hold:

(a) *Every eigenvalue of A is contained in S.*

(b) *If 1 is an eigenvalue of A, then the dimension of the eigenspace corresponding to 1 equals the multiplicity of 1 as an eigenvalue of A.*

Existence of Limits (cont.)

Theorem (5.14)

Let $A \in M_{n\times n}(C)$. $\lim_{m\to\infty} A^m$ *exists if*

(a) *Every eigenvalue of* A *is contained in* S.

(b) A *is diagonalizable.*