Math 4377/6308 Advanced Linear Algebra
5.3 Matrix Limites and Markov Chains

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5.3 Matrix Limits and Markov Chains

- Matrix Limits
- Existence of Limits
**Definition**

Let $L, A_1, A_2, \cdots$ be $n \times p$ matrices with complex entries. The sequence $A_1, A_2, \cdots$ is said to converge to the limit $L$ if

$$\lim_{m \to \infty} (A_m)_{ij} = L_{ij} \text{ for all } 1 \leq i \leq n \text{ and } 1 \leq j \leq p.$$ 

If $L$ is the limit of the sequence, we write $\lim_{m \to \infty} A_m = L$.

**Theorem (5.12)**

Let $A_1, A_2, \cdots$ be a sequence of $n \times p$ matrices with complex entries that converges to $L$. Then for any $P \in M_{r \times n}(C)$ and $Q \in M_{p \times s}(C)$,

$$\lim_{m \to \infty} PA_m = PL \text{ and } \lim_{m \to \infty} A_m Q = LQ.$$
Corollary

Let $A \in M_{n \times n}(C)$ be such that $\lim_{m \to \infty} A^m = L$. Then for any invertible $Q \in M_{n \times n}(C)$,

$$\lim_{m \to \infty} (QAQ^{-1})^m = QLQ^{-1}.$$
5.3

Existence of Limits

Consider the set consisting of the complex number 1 and the interior of the unit disk: \( S = \{ \lambda \in \mathbb{C} : |\lambda| < 1 \text{ or } \lambda = 1 \} \).

Theorem (5.13)

Let \( A \) be a square matrix with complex entries. Then \( \lim_{m \to \infty} A^m \) exists if and only if both of the following hold:

(a) Every eigenvalue of \( A \) is contained in \( S \).
(b) If 1 is an eigenvalue of \( A \), then the dimension of the eigenspace corresponding to 1 equals the multiplicity of 1 as an eigenvalue of \( A \).
Existence of Limits (cont.)

**Theorem (5.14)**

Let \( A \in M_{n \times n}(C) \). \( \lim_{m \to \infty} A^m \) exists if

(a) Every eigenvalue of \( A \) is contained in \( S \).
(b) \( A \) is diagonalizable.