# Math 4377 (15549) / 6308 (14674) (2015 Spring) Advanced Linear Algebra<sup>\*</sup>

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## Prerequisites

Math 2331 or equivalent, and at least 3 hours of 3000 level mathematics.

## Textbook

*Linear Algebra, fourth Edition*, by S.H. Friedberg, A.J Insel, L.E. Spence, Prentice Hall, ISBN 0-13-008451-4

#### **Course content**

Linear Algebra, rich in applications within mathematics and to many other disciplines, is potentially the most interesting and worthwhile undergraduate mathematics course you will complete. For many of you this is the first course to begin bridging the gap between concrete computations and abstract reasoning. Later in your career, computers will do the calculations, but you will have to choose the calculations, know how to interpret the results, and then explain the results to other people. Understanding the notions of vector spaces, linear (in)dependence, dimension, and linear transformations will help you make sense of matrix manipulations at a deeper level, clarifying the underlying structure.

We will begin with the basic theory of vector spaces and subspaces, linear dependence and independence, bases and dimension (Chapter 1). We will then discuss

<sup>\*</sup>This course syllabus provides a general plan for the course; deviations may be necessary.

linear transformations and their connection with matrices. (Chapter 2). In Chapter 3 we will discuss systems of linear equations, Gaussian elimination, and the connection between matrix rank and solvability of a system. Next we will discuss the determinant of an  $n \times n$  matrix, beginning with the properties of the determinant. We will use the properties to derive several methods for computing a determinant. Finally we will discuss eigenvalues, eigenvectors, and diagonalization of an  $n \times n$  matrix (Chapter 5).

Students will be expected to turn in homework weekly. Some of the homework will involve the writing of proofs, which will be graded for clear and logical thinking that is expressed with correct grammar. There will also be five tests (one per chapter), two midterm exams and a final exam.

Students who wish to learn more of this subject may enroll in Math 4378 for the Summer/Spring Semester.

#### Schedule

The following is a tentative schedule, that I will update throughout the semester. Generally we will cover one section per class. Lecture notes are posted section by section on the course homepage. I recommend you print them out or review them before or after lecture, allowing you to focus on the material rather than copying things down.

We will cover sections  $\S1.2-\S1.6$ ,  $\S2.1-\S2.5$ ,  $\S3.1-\S3.4$ ,  $\S4.1-\S4.5$ ,  $\S5.1-\S5.2$ 

- Lecture 1 (1/20): 1.2 Vector Spaces Key concepts: vector space axioms; row vectors, column vectors, matrices; functions, polynomials. Reading: Sections 1.1 & 1.2, Appendix C.
- Lecture 2 (1/22): 1.3 Subspaces Key concepts: subspace characterizations; intersection of subspaces; transpose of matrix, symmetric and skew- or anti-symmetric matrices; trace of matrices, diagonal matrices; upper/lower triangular matrices. Reading: Section 1.3.
- Lecture 3 (1/27): 1.4 Linear combinations and systems of linear equations. Key concepts: linear combinations; solving systems by row reductions; span of a subset; generating sets. Reading: Section 1.4, 1.5.
- Lecture 4 (1/29): 1.5 Linear dependence and independence. Key concepts: linear dependence and independence; Properties of Linear

Dependence and Linear Independence. Reading: Section 1.5, 1.6.

- Lecture 5 (2/3): 1.6 Bases and dimension Key concepts: finite bases; constructing generating sets; finding bases; Replacement Theorem; dimension. Reading: Section 1.6.
- Review & Test 1 (2/5)
- Lecture 6 (2/10): 2.1 Linear transformations. Key concepts: maps of sets; linear maps of vector spaces; kernels and images; nullity and rank, Dimension Theorem. Reading: Section 2.1.
- Lecture 7 (2/12): 2.2 Properties of linear transformations, Matrices. Key concepts: injective, surjective, bijective maps of sets; isomorphisms of vector spaces; coordinates with respect to a basis; matrices with respect to bases; the vector space of linear transformations. Reading: Section 2.1, 2.2.
- Lecture 8 (2/17): 2.3 Composition of linear transformations Key concepts: compositions of maps; basic properties of compositions; multiplication of matrices. Reading: Section 2.3
- Lecture 9 (2/19): 2.4 Invertibility and Isomorphisms. Key concepts: isomorphisms and inverses; every finite dimensional vector space is isomorphic to coordinate space. Reading: Section 2.4.
- Lecture 10 (2/24): 2.5 Change of bases, 2.6 Dual spaces Key concepts: change of coordinate matrices; similar matrices; dual spaces and dual bases; transposes Reading: Section 2.5.
- Review & Test 2 (2/26)
- Midterm Exam 1 (3/3)
- Lecture 11 (3/5): 3.1 Elementary matrix operations. Key concepts: elementary matrices and operations. Reading: Section 3.1

- Lecture 12 (3/10): 3.2 Rank and Inverses Key concepts: rank of matrix; simplifying matrices; calculating inverses. Reading: Section 3.2.
- Lecture 13 (3/12): 3.3 Solving systems of linear equations. Theoretical. Key concepts: consistent and inconsistent systems; homogeneous and nonhomogeneous systems; relation of solutions and null space; criteria for existence of solutions. Reading: Section 3.3.
- Lecture 14 (3/24): 3.4 Solving systems of linear equations. Computational. Key concepts: Gaussian elimination and reduced row echelon form; solving systems of linear equations in reduced row echelon form. Reading: 3.4.
- Review & Test 3 (3/26)
- Lecture 15 (3/31): 4.1 Determinants of Order 2. Key concepts: geometry of determinants of 2x2-matrices; linearity with respect to fixed row. Reading: Section 4.1, 4.2.
- Lecture 16 (4/2): 4.2 Determinants of Order n. Key concepts: geometry of determinants of nxn-matrices; inductive definition of determinants in general; linearity with respect to fixed row; effect of row operations. Reading: Section 4.2.
- Lecture 17 (4/7) 4.3 Properties of Determinants Key concepts: determinant of product of matrices is product of determinants; determinant is nonzero if and only if matrix is invertible; determinants of elementary matrices; determinant of transpose; effect of column operations. Reading: Sections 4.3, 4.4
- Review & Test 4 (4/9)
- Lecture 18 (4/14): 5.1 Eigenvalues and eigenvectors. Key concepts: eigenvectors and eigenvalues; diagonalizability; characteristic polynomial; eigenvalues equal roots of characteristic polynomial. Reading: Section 5.1.
- Lecture 19 (4/16): 5.2 Diagonalizability. Key concepts: algorithm to check diagonalizability; split or non-split polynomials; multiplicity of roots of characteristic polynomial; dimension of

eigenspace. Reading: Section 5.2.

- Lecture 20 (4/21): 5.3 Matrix Limites and Markov Chains Key concepts: Matrix Limits, Existence of Limits Reading: Section 5.3.
- Test 5 (4/23)
- Review (4/28)
- Midterm Exam 2 (4/30)
- Final Exam (Tue 5/12; 11AM 2PM)

## Withdrawal

- Wednesday February 4, 2015: the last day to drop a course or withdraw without receiving a grade.
- Monday April 6, 2015: the last day to drop a course or withdraw with a 'W'.

#### Academic Honesty

Plagiarism and cheating are serious offenses. The University policies on scholastic dishonesty will be strictly enforced (see Catalog/Student Handbook).

## **Attendance Policy**

A student is considered present only if he/she has arrived on time and remains until the class is dismissed. Coming to class late or leaving early is disruptive and thus discouraged. The instructor may drop a student for excessive absences (see Catalog/Student Handbook). Those who have excellent attendance but are on a grade borderline will get extra consideration at the end of the class.

## Cell Phones and Electronic Devices

During class and exam periods, all cell phones and other electronic devices must be turned off and kept in a secure location away from the students immediate view. The use of laptop computers is only permitted if students are using the computers to take notes or for purposes related to the class.

#### **Exam Dates**

As given in the Schedule section, there will be two one-hour in class exams and a cumulative final on the dates

Midterm 1 ( $\S1.2-\S2.5$ )	Tuesday, March 3
Midterm 2 ( $\S3.1 - \S5.2$ )	Thursday, April 30
Final (Cumulative)	Tuesday, May 12, 11am – 2pm

Books and notes will not be allowed on all exams. There will be no make-up exams except in an extreme verifiable emergency. Absence from the final exam will result in a grade of F for the course unless arrangements are made prior to its administration

## Grading

There will be a total of 1000 points possible for this course. The points are distributed as follows

Two midterm exams	200 = 100 + 100
Final Exam	400
10 Highest out of 12 HW Assignments	200 = (25 + 25 + 25 + 25 + 25 + 25 + 25 + 25
4 Highest out 5 Tests	200 = (40 + 40 + 40 + 40)*5/4

Your total accumulated points will determine your letter grade

Percentage Points	Letter Grade
93% - 100~%	А
90% - 92.9~%	A-
87% - 89.9~%	B+
83% - 86.9~%	В
80% - 82.9~%	B-
77%-79.9%	C+
73%-76.9~%	$\mathbf{C}$
70%-72.9~%	C-
67%-69.9~%	D+
63%-66.9~%	D
60%-62.9~%	D-
0%-59.9%	$\mathbf{F}$

Tal	Table 1: Homework				
	Assig	ned Exercises			
Homework 1	§1.2	6, 8, 13, 16, 17, 21.			
Due on $1/29$	$\S{1.3}$	8, 9, 19, 23, 29, 30.			
Homework 2	§1.4	3, 5, 10, 12, 15, 17.			
Due on $2/3$	$\S{1.5}$	3, 9, 11, 12, 13, 17.			
Homework 3	<b>§1</b> 6	4 0 17 00 00 04			
Due on $2/10$	$\S1.6$	4, 9, 17, 20, 26, 34.			
Homework 4	$\S{2.1}$	6, 11, 12, 20, 24, 35.			
Due on $2/19$	$\S2.2$	2, 6, 8, 10, 12, 16.			
Homework 5	$\S{2.3}$	2, 10, 12, 13, 16, 23.			
Due on $2/26$	$\S2.4$	2, 3, 6, 7, 15, 17.			
Homework 6	80 K	2 4 7 0 12 14			
Due on $3/3$	$\S2.5$	2, 4, 7, 9, 12, 14.			
Homework 7	821	2, 4, 5, 7, 10, 12.			
Due on $3/10$	$\S{3.1}$				
Homework 8	$\S{3.2}$	3, 5, 6, 7, 14, 17.			
Due on $3/24$	$\S{3.3}$	2, 3, 8, 10, 12, 14.			
Homework 9	§3.4	1, 2, 4, 7, 10, 12.			
Due on $3/31$	§4.1	2, 3, 4, 7, 9, 12			
Homework 10	§4.2	1, 3, 5, 9, 14, 23			
Due on $4/7$	§4.3	4, 9, 11, 13, 15, 21.			
Homework 11	85 1	5 9 0 19 14 90			
Due on $4/14$	$\S{5.1}$	5, 8, 9, 12, 14, 20.			
Homework 12	<u>የደ ዓ</u>	2 6 7 19 12 18			
Due on $4/21$	$\S{5.2}$	3, 6, 7, 12, 13, 18.			

#### Homework

You may, with impunity, submit up to two assignments up to one class period (not one week) beyond their due date. Subsequent submissions will incur penalties in increments of 10%. Homework submitted later than one class period beyond its due date will not be accepted without a written excuse. Homework scores can not be changed one week after they have been returned. You must staple your work sheets in the upper left hand corner, and make sure that all information (name, etc.) is provided on every page. Any deviation from these instructions will result in a grade of ZERO. Only a subset of the problems collected will be grade. You are encouraged to discuss homework with your classmates. However, you are expected to individually write up your solutions.