Math 6371-13338 (Spring 2013):
Numerical Analysis *

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Texts


Brief Description

This is the second semester of a two semester course. The focus in this semester is on approximation theory and numerical analysis of both ordinary and partial differential equations. The applications of approximation theory to interpolation, Fourier analysis, numerical differentiation and Gaussian integration will be addressed. The concepts of consistency, convergence, stability for the numerical solution of ODEs will be discussed. Other topics covered include multistep and Runge-Kutta methods; finite difference and finite elements techniques for the Poisson equation; and a variety of algorithms to solve large, sparse algebraic systems.

*This syllabus contains important information about this course to which you will need to refer from time to time.*
Course Outline, Homework, and Exam Dates


• **4 Interpolation and Approximation**
  
  − 4.1 The Interpolation Problem
    * 4.1.1 Introduction, 4.1.2 Bases for Polynomial Interpolation, 4.1.3 Conditioning of Polynomial Interpolation
  
  − 4.2 Interpolation Formulas and Algorithms
    * 4.2.1 Newtons Interpolation Formula, 4.2.3 Barycentric Lagrange Interpolation, 4.2.4 Iterative Linear Interpolation, 4.2.6 The Runge Phenomenon
  
  − 4.3 Generalizations and Applications
    * 4.3.1 Hermite Interpolation, 4.3.4 Multivariate Interpolation
  
  − 4.4 Piecewise Polynomial Interpolation
    * 4.4.1 Bernstein Polynomials and Bzier Curves, 4.4.2 Spline Functions, 4.4.3 The B-Spline Basis

• Review Questions I: 4.1[1,2,3,4,5], 4.2[1,2,3,4,5,6,8], 4.3[1,2,6], 4.4[1,2,3,5]

• Problems and Computer Exercises I: 4.1[1], 4.2[1,2,4,5], 4.3[1,2,4,5], 4.4[2,3,4,5,6,7]
  
  − 4.5 Approximation and Function Spaces
    * 4.5.3 Inner Product Spaces and Orthogonal Systems, 4.5.4 Solution of the Approximation Problem, 4.5.5 Mathematical Properties of Orthogonal Polynomials
  
  − 4.6 Fourier Methods
    * 4.6.1 Basic Formulas and Theorems, 4.6.2 Discrete Fourier Analysis,
  
  − 4.7 The Fast Fourier Transform
    * 4.7.1 The Fast Fourier Algorithm, 4.7.2 Discrete Convolution by FFT

• Review Questions II: 4.5[7], 4.6[1], 4.7[1]

• Problems and Computer Exercises II: 4.5[6,18], 4.6[10], 4.7[2]

• **5 Numerical Integration**

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1The dates of exams are subject to change. Changes will be announced in class.
- 5.1 Interpolatory Quadrature Rules
  * 5.1.1 Introduction, 5.1.2 Treating Singularities, 5.1.3 Some Classical Formulas, 5.1.4 Superconvergence of the Trapezoidal Rule
- 5.2 Integration by Extrapolation
  * 5.2.1 The Euler-Maclaurin Formula, 5.2.2 Romberg's Method, 5.2.3 Oscillating Integrands
- 3.4 Acceleration of Convergence
  * 3.4.1 Introduction, 3.4.2 Comparison Series and Aitken Acceleration, 3.4.3 Euler's Transformation, 3.4.5 Euler-Maclaurin's Formula, 3.4.6 Repeated Richardson Extrapolation
- 5.3 Quadrature Rules with Free Nodes
  * 5.3.1 Method of Undetermined Coefficients, 5.3.2 Gauss-Christoffel Quadrature Rules, 5.3.4 Matrices, Moments, and Gauss Quadrature


- Review Questions III: 5.1.3, 5.1.5, 5.2.2, 5.2.4, 3.4.5, 5.3.2, 5.3.4
- Problems and Computer Exercises III: 5.1.10, 5.1.16, 5.2.2, 5.2.7, 3.4.8, 5.3.3, 5.3.11

**EXAM I (March 19)**


**Part I Ordinary differential equations**

- 1. Euler’s method and beyond
  * 1.1 Ordinary differential equations and the Lipschitz condition
  * 1.2 Euler’s method
  * 1.3 The trapezoidal rule
  * 1.4 The theta method
- 2. Multistep methods
  * 2.1 The Adams method
  * 2.2 Order and convergence of multistep methods
  * 2.3 Backward differentiation formulae
\begin{itemize}
  \item 3. RungeKutta methods
      \begin{itemize}
          \item 3.2 Explicit RungeKutta schemes
          \item 3.3 Implicit RungeKutta schemes
      \end{itemize}
  \item 4. Stiff equations
      \begin{itemize}
          \item 4.1 What are stiff ODEs?
          \item 4.2 The linear stability domain and A-stability
          \item 4.3 A-stability of RungeKutta methods
      \end{itemize}
\end{itemize}

\textbf{Part II: The Poisson equation}

\begin{itemize}
  \item 8. Finite difference schemes
      \begin{itemize}
          \item 8.1 Finite differences
          \item 8.2 The five-point formula for $\nabla^2 u = f$
      \end{itemize}
  \item 9. The finite element method
      \begin{itemize}
          \item 9.1 Two-point boundary value problems
          \item 9.2 A synopsis of FEM theory
          \item 9.3 The Poisson equation
      \end{itemize}
  \item 12. Classical iterative methods for sparse linear equations
      \begin{itemize}
          \item 12.1 Linear one-step stationary schemes
          \item 12.2 Classical iterative methods
      \end{itemize}
\end{itemize}

\textbullet Problems and Computer Exercises IV: 1.2, 2.3, 3.4, 4.2

\textbullet **EXAM II (May 2)**