1. Consider the initial value problem
\[ y' = -2y + te^{3t}, \quad 0 \leq t \leq 1, \quad y(0) = 0. \] (1)

(a) Use Euler’s method with \( h = 0.5 \) to approximate the solution to equation (1).
(b) The exact solution to the initial value problem (1) is
\[ y(t) = \frac{1}{5} te^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t} \]
Determine an error bound for the approximation obtained in (a).
(c) Use Taylor’s method of order two with \( h = 0.5 \) to approximate the solution to equation (1).
(d) Use the modified Euler method with \( h = 0.5 \) to approximate the solution to equation (1).
(e) Use the Runge-Kutta method of order four with \( h = 0.5 \) to approximate the solution to equation (1).

2. Consider the following Runge-Kutta method
\[
\begin{align*}
&w_0 = y_0, \\
&w_i = y_i, \quad \text{for } i = 0, 1, \ldots, N - 1, \\
&k_1 = hf(t_i, w_i), \\
&k_2 = hf(t_i + \alpha h, w_i + \beta k_1) \\
&w_{i+1} = w_i + a_1 k_1 + a_2 k_2
\end{align*}
\]

(a) Show that the above Runge-Kutta method cannot have local truncation error \( O(h^3) \) for any choice of constants \( a_1, a_2, \alpha \) and \( \beta \).
(b) Show that the above Runge-Kutta method is of order 2 if, for any \( \alpha \),
\[ \beta = \alpha, \quad a_1 = 1 - \frac{1}{2\alpha}, \quad a_2 = \frac{1}{2\alpha}. \]
(c) Show that by choosing \( \alpha = 1 \) in (b), we obtain the modified Euler method.
(d) Show that by choosing \( \alpha = \frac{1}{2} \) in (b), we obtain the midpoint method.

3. Derive the Adams-Bashforth two step method by using the Lagrange form of the interpolating polynomial.