Numerical Ordinary Differential Equations

MATH 6372-11153 (Fall 2002)

Time:	5:30-7:00 p.m. MW
Room:	201-AH
Instructor:	J.W. He
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Office hours:	1:00-2:00 p.m. Tuesday or by appointment

Text Book

Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations, Uri M. Ascher and Linda R. Petzold, SIAM, 1998, ISBN 0-89871-412-5. List Price $43.50 / SIAM Member Price $30.45.

Course Description

Introduction to ordinary differential equations; theory of general linear methods; stability analysis and linear multistep methods; Runge-Kutta methods and extrapolation methods; stiffness and nonlinear methods; Hamiltonian systems and symplectic methods; numerical methods for singular perturbation problems; numerical methods for differential-algebraic equations

The course is aimed at a thorough understanding of the issues and methods for practical computation while avoiding an extensive theorem-proof type of exposition. It also addresses reasons why existing software succeeds or fails.

Prerequisites

Numerical analysis (MATH 4364, Math 6370) and an undergraduate ODE course (Math 3331).

Course Policies

Homework: Total 6 Assignments (one per chapter). Class will be divided to 3 groups, denoted by A, B and C. There will be in average 4 problems per group per assignment.

Exam: None

Reference Books

1. K. Brenan, S. Campbell and L. Petzold (1996): Numerical Solution of Initial Value Problems in Differential-Algebraic Equations. SIAM, Philadelphia, 2nd edition.

2. J.C. Butcher (1987): The Numerical Analysis of Ordinary Differential Equations. Wiley, New York.

3. C.W. Gear (1973): Numerical Initial Value Problems in Ordinary Differential Equations. Prentice-Hall, Englewood Cliffs, NJ.

4. E. Griepentrog and R. Marz (1986): Differential-Algebraic Equations and Their Numerical Treatment. Teubner, Leipzig.

5. E. Hairer, S.P. Norsett and G. Wanner (1993): Solving Ordinary Differential Equations I. Nonstiff Problems. Springer Series in Comput. Math., vol. 8, 2nd edition.

6. E. Hairer and G. Wanner (1996): Solving Ordinary Differential Equations II. Stiff and Differential-Algebraic Problems. Springer Series in Comput. Math., vol. 14, 2nd edition.

7. E. Hairer, Ch. Lubich and G. Wanner (2002): Geometric Numerical Integration Structure-Preserving Algorithms for Ordinary Differential Equations. Springer Series in Computational Mathematics 31, Springer-Verlag.

8. P. Henrici (1962): Discrete Variable Methods in Ordinary Differential Equations. John Wiley & Sons.

9. A. Iserles (1996): A First Course in the Numerical Analysis of Differential Equations. Cambridge Texts in Applied Mathematics, Cambridge University Press.

10. J.D. Lambert (1991): Numerical Methods for Ordinary Differential Equations. John Wiley & Sons.

11. R. Marz (1992): Numerical Method for Differential-Algebraic Equations. Acta Numerical, 1:141-198.

12. R.M.M. Mattheij and J. Molnaar (1996): Ordinary Differential Equations in Theory and Practice. Wiley, Chichester, UK.

13. J.M. Sanz-Serna and M.P. Calvo (1994): Numerical Hamiltonian Problems. Chapman and Hall, London.

14. L.F. Shampine (1994): Numerical Solution of Ordinary Differential Equations. Chapman and Hall, London.

15. J. Stoer and R. Bulirsch (1993): Introduction to Numerical Analysis. Springer Texts in Appl. Math., vol. 12, 2nd edition.

16. A.M. Stuart and A.R. Humphries (1996): Dynamical Systems and Numerical Analysis. Cambridge Univ. Press.

Course Outline[1]

Chapter 1 (Two Lectures)

Topics: Ordinary Differential Equations: IVPs, DAEs, and Applications

Dates: Aug. 26 and Sept. 4

References: Mattheij and Molnaar (1996), Stoer and Bulirsch (1993), Hairer, Norsett and Wanner (1993).

Homework: None.

Chapter 2 (Two Lectures)

Topics: On Problem Stability: Definitions, Linear Systems, Nonlinear Problems, Hamiltonian Systems

Dates: Sept. 9 and 11

References: Hairer, Norsett and Wanner (1993), Mattheij and Molnaar (1996), Stuart and Humphries (1996), Henrici (1962).

Homework: Problems 2.1-2.4 (Groups A, B and C), Due Date: Sept. 16.

Chapter 3 (Four Lectures)

Topics: Basic Methods, Basic Concepts: Convergence, Accuracy, Consistency, 0-Stability, A-Stability, Stiffness

Dates: Sept. 16, 18, 23 and 25

References: Henrici (1962), Gear (1973), Lambert (1991), Shampine (1994), Butcher (1987), Stuart and Humphries (1996), Mattheij and Molnaar (1996).

Homework: Problems 3.1, 3.4, 3.7, 3.10 (Group A), Problems 3.2, 3.5, 3.8, 3.11 (Group B) and Problems 3.3, 3.6, 3.9 (Group C), Due Date: Sept. 30.

Chapter 4 (Six Lectures)

Topics: One-Step Methods: RK Methods, Convergence, Stability and Order, Error Estimation and Control, Implicit RK Methods.

Dates: Sept. 30, Oct. 2, 7, 9, 14 and 16

References: Hairer, Norsett and Wanner (1993), Butcher (1987), Lambert (1991), Gear (1973), Hairer and Wanner (1996), Shampine (1994).

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[1]This syllabus is subject to change. Changes will be announced in class.

Homework: Problems 4.1, 4.4, 4.7, 4.10, 4.13, 4.16, 4.19 (Group B), Problems 4.2, 4.5, 4.8, 4.11, 4.14, 4.17 (Group C) and Problems 4.3, 4.6, 4.9, 4.12, 4.15, 4.18 (Group A), Due Date: Oct. 21.

Chapter 5 (Five Lectures)

Topics: Linear Multistep Methods: Convergence, Stability and Order, Implicit Multistep Methods, Implementation

Dates: Oct. 21, 23, 28, 30 and Nov. 4

References: Hairer, Norsett and Wanner (1993), Hairer and Wanner (1996), Lambert (1991), Gear (1973), Shampine (1994), Henrici (1962).

Homework: Homework assignment: Problems 5.1, 5.4, 5.7, 5.10, 5.13 (Group C), Problems 5.2, 5.5, 5.8, 5.11 (Group A) and Problems 5.3, 5.6, 5.9, 5.12 (Group B), Due Date: Nov. 6.

Chapter 9 (Four Lectures)

Topics: More on Differential-Algebraic Equations: Index and Mathematical Structure, Index Reduction and Stabilization, Modeling

Dates: Nov. 6, 11, 13 and 18

References: Brenan, Campbell and Petzold (1996), Hairer and Wanner (1996), Hairer, Lubich and Wanner (2002), Griepentrog and Marz (1986), Marz (1992).

Homework: Problems 9.1, 9.4, 9.7, 9.10 (Group A), Problems 9.2, 9.5, 9.8, 9.11 (Group B) and Problems 9.3, 9.6, 9.9 (Group C), Due Date: Nov. 20.

Chapter 10 (Five Lectures)

Topics: Numerical Methods for DAEs: Direct Discretization Methods, Methods on Manifolds

Dates: Nov. 20, 25, Dec. 2, 4 and 9

References: Hairer, Lubich and Wanner (2002), Hairer and Wanner (1996), Brenan, Campbell and Petzold (1996), Griepentrog and Marz (1986), Marz (1992).

Homework: Homework assignment: Problems 10.1, 10.4, 10.7, 10.10, 10.13, 10.16 (Group B), Problems 10.2, 10.5, 10.8, 10.11, 10.14 (Group C) and Problems 10.3, 10.6, 10.9, 10.12, 10.15 (Group A), Due Date: Dec. 11.