Math 1431 - 15825

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Office Hours: 11-noon MWF

Course Homepage
http://www.math.uh.edu/~jmorgan/Math1431

Reminders:
- Test 1, Practice Test 1, Quiz 1 are all available online.
- EMCF01 was due this morning at 9am.
- EMCF02 is due on Friday morning at 9am.
- Homework 1 is posted and due next Wednesday.
- Written Quiz 1 will be given in lab/workshop on Friday.
- Purchase your Access Code NOW from the UH Bookstore, and input it on CourseWare at http://www.easa.uh.edu. Pick up your Popper Forms by the end of next week.

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Read the Syllabus
Access all Online Quizzes, Practice Test 1, Test 1, answer sheets for EMCF's and the Discussion Board on CourseWare at http://www.easa.uh.edu. All practice tests count as online quizzes, and all online quizzes and online tests expire at 11:59pm on the stated date.

Note: The test dates listed below are subject to change.

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<thead>
<tr>
<th>Sunday</th>
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<tr>
<td>August 28</td>
<td>Practice Test 1</td>
<td>Test 1, and Online Quizzes are Open</td>
<td>Name, Test 1 is online, and from HW 1 - 2 answers</td>
<td>EMCF1 Due Online or Back Quiz at Lab/Workshop</td>
<td>September 1</td>
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Note: There are only 2 attempts for online Test 1.

Limits - Basic Ideas and Review

The limit of \( f(x) \) as \( x \) approaches \( a \) is given by the value (if it exists) we expect from \( f(x) \) as \( x \) approaches \( a \).

\[
\lim_{x \to a} f(x) = L
\]

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The limit of $f(x)$ as $x$ approaches $a$ is $L$ if and only if $f(x)$ can be made arbitrarily close to $L$ by making $x \neq a$ sufficiently close to $a$.

The limit of $f(x)$ as $x$ approaches $a$ is $L$ if and only if

$$\lim_{x \to a} f(x) = L.$$

One-Sided Limits, in words...

The limit of $f(x)$ as $x$ approaches $a$ from the left is $L$ if and only if $f(x)$ can be made arbitrarily close to $L$ by making $x < a$ sufficiently close to $a$.

The limit of $f(x)$ as $x$ approaches $a$ from the right is $L$ if and only if $f(x)$ can be made arbitrarily close to $L$ by making $x > a$ sufficiently close to $a$.

The limit of $f(x)$ as $x$ approaches $a$ from the left is $L$, equivalent to the notation $\lim_{x \to a^-} f(x) = L$.

The limit of $f(x)$ as $x$ approaches $a$ from the right is $L$, equivalent to the notation $\lim_{x \to a^+} f(x) = L$.

The Fundamental Relationship Between Left Hand Limits, Right Hand Limits, and Limits

$$\lim_{x \to a^-} f(x) = L$$

if and only if

$$\lim_{x \to a^+} f(x) = L$$

if and only if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$
Suppose \( g(x) = \begin{cases} 
-x + 2, & x < 1 \\
3, & x = 1 \\
3x - 2, & x > 1 
\end{cases} \). Find \( \lim_{x \to 0^-} g(x) \) and \( \lim_{x \to 0^+} g(x) \).

\[
\lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} g(x) = 2
\]

Intuitive Limits From Formulas...

\[
\lim_{x \to 2} \left(2x^2 - 1\right) = 7
\]

\[
\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x-1)}{(x-2)} = \lim_{x \to 2} (x-1) = 1
\]

\[
\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x-1)}{x - 2} = \lim_{x \to 2} (x-1) = 1
\]
\[ \lim_{x \to 2} f(x) \quad \text{where} \quad f(x) = \begin{cases} 2x - 1 & x < 2 \\ 6 - x^2 & x \geq 2 \end{cases} \]

\[ \lim_{x \to 2^-} f(x) = \lim_{x \to 2} (2x - 1) = 3 \]

\[ \lim_{x \to 2^+} f(x) = \lim_{x \to 2} (6 - x^2) = 2 \]

\[ (a-b)(a+b) = a^2 - b^2 \]

\[ \lim_{x \to 1} \frac{x^2 - 1}{x-1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)} \]

\[ = \lim_{x \to 1} (x+1) \]

\[ = 1 \]

\[ \lim_{x \to 0} \frac{2}{2x} = \frac{1}{2} \]

\[ \text{our limit} \]

\[ \text{What does it mean to say that } x \text{ is within } .01 \text{ of } 12 \text{ but not equal to } 12 \]

\[ |x-12| < .01 \text{ and } x \neq 12 \]

\[ 0 < |x-12| < .01 \]

"x is within .01 of 12, but not equal to 12"

\[ 0 < |x-7| < .001 \]

"x is within .001 of 7 but not equal to 7"
What does it mean to say that $f(x)$ is within $\varepsilon$ of $L$?

$\lim_{x \to a} f(x) = L$ 

$\varepsilon > 0$

aproaches $a$ if and only if $f(x)$ can be made arbitrarily close to $L$ by making $x \neq a$ sufficiently close to $a$.

Let $\varepsilon > 0$. Then there is a value $\delta > 0$ so that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

"$f(x)$ is within $\varepsilon$ of $L$"

What does it mean to say that $(x$ is within $\delta$ of $c)$ but not equal to $c$?

$0 < |x - c| < \delta$

"$x$ is within $\delta$ of $c$ but not equal to $c$"

Let's use this language to discuss limits...

Give the largest $\delta$ that works with $\varepsilon = 0.1$ for the limit

$\lim_{x \to 1} (1 - 2x) = 3$

$f(x) = 1 - 2x$

$L = 3$, $a = 1$

$\varepsilon = 0.1$

Let $\varepsilon > 0$. Then there is a value $\delta > 0$ so that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

When $0 < |x - 1| < \delta$

$0 < |x + 1| < 0.5$

Use $\delta = 0.5$.
Prove that \( \lim_{x \to 1} (1-2x) = 3 \).

Let \( \epsilon > 0 \). Then let \( \delta = \frac{\epsilon}{2} \). If \( |x-1| < \delta \), then

\[
|1-2(x+1)| = 2|x+1| < 2\cdot\frac{\epsilon}{2} = \epsilon.
\]

Then

\[
|1-(2x+3)| < \epsilon \Rightarrow \lim_{x \to 1} (1-2x) = 3.
\]