Math 1431 - 15825

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Course Homepage
http://www.math.uh.edu/~jmorgan/Math1431
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Reminders:
• Homework 1 is due next Wednesday in lab/workshop.
• There is a quiz today in lab/workshop.
• Test 1 and Practice Test 1 are due next Wednesday. They are available online at http://www.casa.uh.edu.
• Quiz 1 is available at http://www.casa.uh.edu.
• EMCF03 is posted, and it is due next Tuesday.
• Poppers start on Monday, September 10th.
• Purchase your Access Code from the UH Bookstore, and then log into this course at http://www.casa.uh.edu and input the code.

Take advantage of the Discussion Board on CourseWare!

Limit Theorems

Uniqueness, sums, differences, products and quotients.

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Uniqueness of the Limit

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} f(x) = M \),
then \( L = M \).

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The Limit of the Sum
If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \), then \( \lim_{x \to c} (f(x) + g(x)) = L + M \).

is the Sum of the Limits
(provided the limits exist)

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The Limit of the Difference
If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \), then \( \lim_{x \to c} (f(x) - g(x)) = L - M \).

is the Difference of the Limits
(provided the limits exist)

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The Limit of the Product
If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \), then \( \lim_{x \to c} f(x) \cdot g(x) = L \cdot M \).

is the Product of the Limits
(provided the limits exist)

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The Limit of the Quotient
If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \), and \( M \neq 0 \), then \( \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M} \).

is the Quotient of the Limits
(provided the quotient exists)

(provided the limits exist)
More on Quotients

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \), and \( L \neq 0 \) and \( M = 0 \), then \( \lim_{x \to c} \frac{f(x)}{g(x)} \) d.n.e.

How Can We Use This Information?
Building Blocks

\( \lim_{x \to c} a = a \)

\( \lim_{x \to c} x = c \)

What is a Polynomial Function?

Anything of the form \( f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \)

For example:

\( f(x) = -2x^3 + x - 1 \)

\( g(x) = 9x^2 - 3x + \frac{1}{2} \)

\( r(x) = 5x - 7 \)

\( S(t) = -8t^5 + t^3 - 17t \)
What is a Rational Function?

A Rational Function is a Quotient of Polynomials

For example

\[ f(x) = \frac{-2x^3 + x - 1}{3x^2 + 1} \]

\[ r(x) = \frac{-2}{3x + 1} \]

\[ G(v) = \frac{v^2 - 3v}{2v^2 + v - 1} \]

Note:

Polynomial and Rational Functions Are Sums, Products and Quotients of Scalars and the independent variable.

Theorem

Let \( p(x) \) be a polynomial and let \( a \) be a real number. Then

\[ \lim_{x \to a} p(x) = p(a) \]

i.e. We can take limit of a polynomial function by simply evaluating the function.
Example: $\lim_{x \to -2} (-5x^4 + 3x^2 + x)$

How do we evaluate

$$\lim_{x \to a} \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials?

Theorem

Let $p(x)$ and $q(x)$ be polynomials, and let $a$ be a real number. Then

$$\lim_{x \to a} \frac{p(x)}{q(x)} = \begin{cases} 
\frac{p(a)}{q(a)}, & \text{if } q(a) \neq 0 \\
\text{d.n.e.,} & \text{if } p(a) \neq 0 \text{ and } q(a) = 0
\end{cases}$$

If $p(a) = q(a) = 0$ then more work is required to determine the limit.

Example: $\lim_{x \to 2} \frac{-2x^3 + x - 1}{3x^2 + 1}$
Example: \( \lim_{x \to 1} \frac{x^3 - 1}{x^2 + x - 2} \)

Example: \( \lim_{x \to -1} \frac{x^3 - 1}{x^2 + x} \)

Example: \( f(x) = 2x^2 - x + 1. \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \)

Example: \( f(x) = \frac{x}{x+1}, \lim_{x \to 1} \frac{f(x) - f(1)}{x-1} = \)
Note: We have learned that whenever $f(x)$ is a polynomial or rational function and $c$ is in the domain of $f(x)$, then

i.e. we can evaluate the limit by evaluating the function?

Question: Are there other functions that have this property?