Math 1431 - 15825

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Course Homepage
http://www.math.uh.edu/~jmorgan/Math1431
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Reminders:
• Homework 1 is due next Wednesday in lab/workshop.
• There is a quiz today in lab/workshop.
• Test 1 and Practice Test 1 are due next Wednesday. They are available online at http://www.casa.uh.edu.
• Quiz 1 is available at http://www.casa.uh.edu.
• EMCF03 is posted, and it is due next Tuesday.
• Poppers start on Monday, September 10th.
• Purchase your Access Code from the UH Bookstore, and then log into this course at http://www.casa.uh.edu and input the code.

Take advantage of the Discussion Board on CourseWare!

Limit Theorems

Uniqueness, sums, differences, products and quotients.

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Uniqueness of the Limit

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} f(x) = M \), then \( L = M \).

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The Limit of the Sum

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \),
then \( \lim_{x \to c} (f(x) + g(x)) = L + M \).

is the Sum of the Limits
(provided the limits exist)

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The Limit of the Difference

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \),
then \( \lim_{x \to c} (f(x) - g(x)) = L - M \).

is the Difference of the Limits
(provided the limits exist)

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The Limit of the Product

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \),
then \( \lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M \).

is the Product of the Limits
(provided the limits exist)

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The Limit of the Quotient

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \),
and \( M \neq 0 \), then \( \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M} \).

is the Quotient of the Limits
(provided the limits exist)

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More on Quotients

If \( \lim_{x \to \infty} f(x) = L \) and \( \lim_{x \to \infty} g(x) = M \), and \( L \neq 0 \) and \( M = 0 \), then \( \lim_{x \to \infty} \frac{f(x)}{g(x)} \) does not exist.

Note: If \( L = 0 \) and \( M = 0 \), then you need to do more work. Recall: \( \frac{0}{0} \) is an indeterminate form.

What is a Polynomial Function?

If the independent variable is \( x \), then a polynomial in \( x \) is an expression that can be built from sums, differences, and products of \( x \) with itself and scalars.

- \( f(x) = -3 \)
- \( g(x) = 2x - 1 \)
- \( F(x) = 3x^3 - 2x + 7 \)

Note: \( G(x) = \frac{3x}{x-1} \) is not a polynomial.

\( H(x) = 3x^3 - 1 \)

Answer

Anything of the form \( f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \)

Assuming \( x \) is the independent variable.

- \( n \) is a positive integer.

For example:
- \( f(x) = -2x^3 + x - 1 \) is a 3rd degree polynomial.
- \( g(x) = 9x^2 - 3x + \frac{1}{2} \) is a 2nd degree.
- \( r(x) = 5x - 7 \) is a 1st degree.
- \( S(t) = -8t^4 + r^3 - 17t \) is a 0th degree.
What is a Rational Function?

A Rational Function is a Quotient of Polynomials

For example

\[ f(x) = \frac{-2x^2 + x - 1}{3x^2 + 1} \quad \text{poly} \]

\[ r(x) = \frac{-2}{3x + 1} \quad \text{poly} \]

\[ G(v) = \frac{v^3 - 3v}{2v^2 + v - 1} \quad \text{poly} \]

\[ H(x) = 2x^3 - 3x + 1 = \frac{2x^3 - 3x + 1}{1} \quad \text{poly} \]

Note: Every polynomial is a rational function.

Theorem

Let \( p(x) \) be a polynomial and let \( a \) be a real number. Then

\[
\lim_{{x \to a}} p(x) = p(a)
\]

i.e. We can take the limit of a polynomial function by simply evaluating the function.

Note:

Polynomial and Rational Functions Are Sums, Products and Quotients of Scalars and the independent variable.
Example: \( \lim_{x \to 2} (-5x^4 + 3x^3 + x) = -70 \)

\[
-5(-2)^4 + 3(-2)^3 + (-2) \\
= -80 + 12 - 2 \\
= -82 + 12 \\
= -70
\]

How do we evaluate \( \lim_{x \to a} \frac{p(x)}{q(x)} \) where \( p(x) \) and \( q(x) \) are polynomials?

Theorem

Let \( p(x) \) and \( q(x) \) be polynomials, and let \( a \) be a real number. Then

\[
\lim_{x \to a} \frac{p(x)}{q(x)} = \begin{cases} 
\frac{p(a)}{q(a)}, & \text{if } q(a) \neq 0 \\
\text{d.n.e.} & \text{if } p(a) = 0 \text{ and } q(a) = 0
\end{cases}
\]

If \( p(a) = q(a) = 0 \) then more work is required to determine the limit.

Example: \( \lim_{x \to 2} \frac{-2x^2 + x - 1}{3x^2 + 1} \)

\[
= \frac{-15}{13}
\]
Example: $\lim_{x \to 1} \frac{x^3 - 1}{x^3 + x - 2}$

Evaluate at $x = 1$.

Get to work!

= $\frac{3}{3} = 1$.

Example: $f(x) = 2x^3 - x + 1$, $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

= $\lim_{h \to 0} \frac{2(4+4h+h^2) - 2 - h - 1}{h}$

= $\lim_{h \to 0} \frac{8h + 2h^2 - h - 1}{h}$

= $\lim_{h \to 0} \frac{2h + 2h}{h}$

= $\lim_{h \to 0} \frac{2h + 2h}{h} = \lim_{h \to 0} \frac{7 + 2h}{1}$

= 7.
Note: We have learned that whenever \( f(x) \) is a polynomial or rational function and \( c \) is in the domain of \( f(x) \), then

\[
\lim_{x \to c} f(x) = f(c)
\]

i.e. we can evaluate the limit by evaluating the function?

**Question:** Are there other functions that have this property?

**Yes.** Any function built from

\[ \sqrt{1 + \frac{1}{x}}, \frac{1}{x}, \tan x \]

provided the expression is defined in an interval containing \( c \).