Math 1431 - 15825
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Notes
- **Poppers** start next Monday! Access Codes must be purchased and entered at www.casa.uh.edu by next Monday.
- Purchase your Popper Forms and Access Code from the Bookstore in the University Center
- **Homework 1** is due today in recitation/workshop.
- **EMCF03** was due at 9am yesterday, and **EMCF04** is due on Friday at 9am.
  Homework 2 will be posted today.
- **Online Quizzes** are available, and **Test 1** and **Practice Test 1** are due tonight at 11:59pm.
- There is a Written Quiz in lab/workshop on Friday.

Recall

Let \( p(x) \) be a polynomial and let \( c \) be a real number. Then
\[
\lim_{x \to c} p(x) = p(c).
\]

Theorem

Let \( p(x) \) and \( q(x) \) be polynomials, and let \( c \) be real numbers. Then
\[
\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{p(c)}{q(c)},
\]
if \( p(c) \neq 0 \) and \( q(c) \neq 0 \). If \( p(c) = q(c) = 0 \) then more work is required to determine the limit.

Note: We have learned that whenever \( f(x) \) is a polynomial or rational function, and \( c \) is in the domain of \( f \), then
\[
\lim_{x \to c} f(x) = f(c)
\]
i.e. we can evaluate the limit by evaluating the function.
What does this say about the graphs of polynomial and rational functions?

"Limit is function value." "In the domain.

This property is known as continuity.

"The function is continuous at $x = c$"

What does it mean to say that $f(x)$ is **continuous** at $x = c$?

Geometrically?

In terms of limits?

1. $f(c)$ exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Polynomials and Rational Functions are Continuous Everywhere They are Defined

**Key point.**

For polynomials: Everywhere

For rational functions: Everywhere the denom $\neq 0$.

**Terminology**

$f(x)$ is **discontinuous** at $x = c$ if and only if $f(x)$ is **not continuous** at $x = c$. 
What are the **Basic Types of Discontinuity**?

1. **Removable discontinuity**.

2. **Jump discontinuity**.

3. **Infinite discontinuity**.

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**Removable Discontinuity at** $x = c$.

**Graph:**

[Graph showing removable discontinuity]

**Def.:** $\lim_{x \to c} f(x)$ exists.

**But** $\lim_{x \to c} f(x) \neq f(c)$.

Either $\infty \neq \infty$ or $f(c)$ does not exist.

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**Jump Discontinuity at** $x = c$.

**Graph:**

[Graph showing jump discontinuity]

**Def.:**

- $\lim_{x \to c^-} f(x)$ exists.
- $\lim_{x \to c^+} f(x)$ exists.

**But** $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$.

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**Infinite Discontinuity at** $x = c$.

**Vertical asymptote**

(at least from one side)

[Graph showing infinite discontinuity]
Example: Discuss the continuity of the following functions.

\[ G(x) = 3x^3 - 2x^2 - 7 \]

\[ f(x) = \frac{x - 1}{|x - 1|} \]

\[ H(x) = \frac{x^2 - 4}{x + 2} \]

\[ g(x) = \frac{x + 2}{x^2 + x - 2} \]

\[ G(x) = 3x^3 - 2x^2 - 7 \]

\[ \text{polynomial} \]

\[ \cdot G(x) \text{ is continuous everywhere.} \]

\[ \cdot g(x) \text{ is continuous everywhere it is defined.} \]

\[ \cdot \text{where is } g(x) \text{ defined?} \]

\[ \cdot \text{anywhere that the denominator is not 0.} \]

\[ \cdot \text{undefined when } x^2 + x - 2 = 0 \]

\[ \cdot (x + 2)(x - 1) = 0 \]

\[ \cdot x = -2 \text{ or } x = 1. \]

\[ \cdot g(x) \text{ is defined and continuous everywhere except } x = -2 \text{ and } x = 1. \]

\[ \cdot \text{What sort of discontinuity occurs?} \]

\[ (-\infty, -2) \cup (-2, 1) \cup (1, \infty) \]

\[ \text{Note: } g(x) = \frac{x + 2}{x^2 + x - 2} \]

\[ \text{is discontinuous at } x = -2 \text{ and } x = 1. \]

\[ \text{Let's classify these:} \]

\[ \text{Rewrite: } g(x) = \frac{x + 2}{x(x + 2)} \]

\[ = \frac{(x + 2)}{(x + 2)(x - 1)} \]

\[ = \frac{1}{x - 1}, \quad x \neq 2 \]

\[ \text{This gives a removable discontinuity at } x = -2 \text{ and an infinite discontinuity at } x = 1. \]
Give values for $A$ and $B$ so that $f(x) = \begin{cases} 4x - 3 & x < -2 \\ 2 & x = -2 \\ x^2 - B & x > -2 \end{cases}$ is continuous.

$x = -2$ is the only possible problem. For continuity at $x = -2$, we need

$f(-2) = \lim_{{x \to -2^-}} f(x) \quad \lim_{{x \to -2^+}} f(x) \quad \text{One-sided limits.}$