Math 1431 - 15825

Jeff Morgan
jmorgan@math.uh.edu
651 PGH
Office Hours: 11:00-Noon MWF

Homepage URLs and Twitter

www.math.uh.edu/~jmorgan/Math1431
tinyurl.com/math1431
@morgancalculus

Twitter

Notes

- **Poppers** start next Monday! Access Codes must be purchased and entered at www.casa.uh.edu by next Monday. ← 12:01 AM
- Purchase your **Popper Forms** and **Access Code** from the Bookstore in the University Center
- **Homework 1** is due today in recitation/workshop.
- **EMCF03** was due at 9am yesterday, and **EMCF04** is due on Friday at 9am. **Homework 2** will be posted today.
- **Online Quizzes** are available, and **Test 1** and **Practice Test 1** are due tonight at 11:59pm.
- There is a **Written Quiz** in lab/workshop on Friday.

Recall

Theorem

Let $p(x)$ be a polynomial and let c be a real number. Then $\lim_{x\to c} p(x) = p(c)$.

Theorem

rational function

Let $p(x)$ and $q(x)$ be polynomials, and let c be a real numbers. Then

$$\lim_{x\to c}\frac{p(x)}{q(x)} = \begin{cases} \frac{p(c)}{q(c)}, & \text{if } \boxed{q(c)\neq 0} \\ \boxed{\text{undefined}}, & \text{if } \boxed{p(c)\neq 0} \text{ and } \boxed{q(c)=0} \end{cases}$$

denom ≠ 0

denom = 0

dne

numer ≠ 0

If $p(c)=q(c)=0$ then more work is required to determine the limit.

" $\frac{0}{0}$ " ind. form.

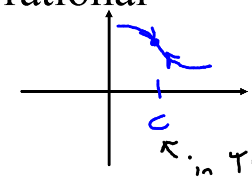
Get to work.

Note: We have learned that whenever $f(x)$ is a polynomial or rational function, and c is in the domain of f, then

Huge → $\lim_{x\to c} f(x) = f(c)$

i.e. we can evaluate the limit by evaluating the function.

What does this say about the graphs of polynomial and rational functions?



"Limit is function value."

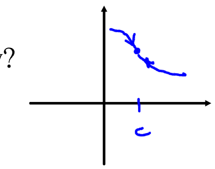
in the domain.

This property is known as continuity.

"the function is continuous at $x = c$"

What does it mean to say that $f(x)$ is continuous at $x = c$?

Geometrically?



In terms of limits?

you get what you expect to get.

1. $f(c)$ exists
2. $\lim\limits_{x \to c} f(x)$ exists
3. $\lim\limits_{x \to c} f(x) = f(c)$.

Defining Properties

Polynomials and Rational Functions are Continuous Everywhere They are Defined

Key point.

Ben

For Polynomials: Everywhere

For Rational Functions: Everywhere the denom $\neq 0$.

Terminology

$f(x)$ is discontinuous at $x = c$
if and only if $f(x)$
is not continuous at $x = c$.

What are the Basic Types of Discontinuity?

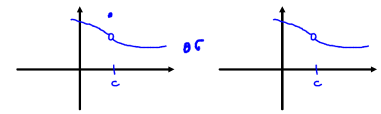
1. Removable discontinuity.

2. Jump discontinuity.

3. Infinite discontinuity.

Removable Discontinuity at $x = c$.

Graph:



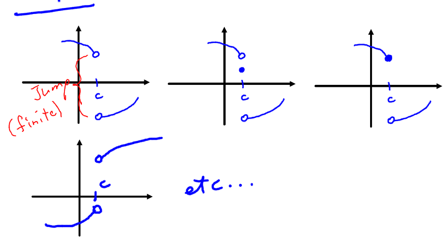
Def: $\lim_{x \to c} f(x)$ exists.

But $\lim_{x \to c} f(x) \neq f(c)$.

Either b/c $f(c)$ dne or b/c the limit and function value do not agree.

Jump Discontinuity at $x = c$.

Graph:



etc...

Def: $\lim_{x \to c^-} f(x)$ exists

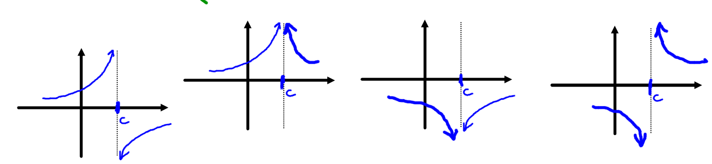
and $\lim_{x \to c^+} f(x)$ exists,

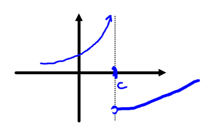
BUT $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$.

Infinite Discontinuity at $x = c$

$\underbrace{\text{Infinite Discontinuity}}_{\text{vertical asymptote}}$

(at least from one side)





etc...

Example: Discuss the continuity of the following functions.

$$G(x) = 3x^3 - 2x^2 - 7$$

$$f(x) = \frac{x-1}{|x-1|}$$

$$H(x) = \frac{|x^2-4|}{x+2}$$ ← see the video.

$$g(x) = \frac{x+2}{x^2+x-2}$$

$$\underbrace{G(x) = 3x^3 - 2x^2 - 7}_{\text{polynomial}}$$

$\therefore$ $G(x)$ is continuous everywhere.

$$g(x) = \frac{x+2}{\underbrace{x^2+x-2}}$$

rational function

$\therefore$ $g(x)$ is continuous everywhere it is defined.

Q: where is $g(x)$ defined?

A: Anywhere that the denom $\neq 0$.

i.e. undefined when $x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2$ or $x=1$.

$\therefore$ $g(x)$ is defined and continuous everywhere except $x=-2$ and $x=1$.

Hm.... what sort of discont occurs?

$(-\infty,-2)\cup(-2,1)\cup(1,\infty)$

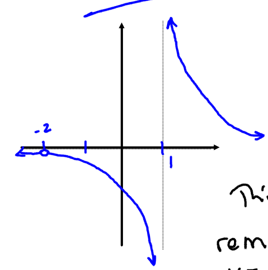
Note: $g(x) = \frac{x+2}{x^2+x-2}$ is discont at $x=-2$ and $x=1$.

Let's classify these:

Rewrite: $g(x) = \frac{x+2}{x^2+x-2}$

$= \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$

$= \frac{1}{x-1}$, $x \neq -2$, $x \neq 1$



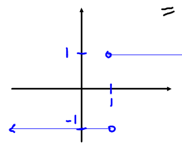
This gives a removable discont at $x=-2$ and an infinite discont at $x=1$.

$$f(x)=\frac{x-1}{|x-1|}$$

Note: $|x-1| = \begin{cases} -(x-1), & x<1 \quad (x-1<0) \\ x-1, & x\geq 1 \quad (x-1>0) \end{cases}$

$$f(x)=\frac{x-1}{|x-1|} = \begin{cases} \frac{x-1}{-(x-1)}, & x<1 \\ \frac{x-1}{x-1}, & x>1 \end{cases}$$

$$= \begin{cases} -1, & x<1 \\ 1, & x>1 \end{cases}$$



$\therefore$ f is continuous everywhere except $x=1$, and f has a jump discont at $x=1$.

$(-\infty,1)\cup(1,\infty)$

Give values for A and B so that

$$f(x)=\begin{cases} Ax-3, & x<-2 \\ 2 & x=-2 \\ x^2-B & x>-2 \end{cases}$$

is continuous.

line

parabola

continuous

$x=-2$ is the only possible problem.

For continuity at $x=-2$, we need

$$\underset{2}{\underbrace{f(-2)}} = \lim_{x\to -2} f(x)$$

One sided limits.

See the video.