Math 1431 - 15825

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Notes

- **Poppers** start next Monday! Access Codes must be purchased and entered at www.casa.uh.edu by next Monday.
- Purchase your **Popper Forms** and **Access Code** from the Bookstore in the University Center
- **Homework 1** is due today in recitation/workshop.
- **EMCF03** was due at 9am yesterday, and **EMCF04** is due on Friday at 9am. **Homework 2** will be posted today.
- **Online Quizzes** are available, and **Test 1** and **Practice Test 1** are due tonight at 11:59pm.
- There is a **Written Quiz** in lab/workshop on Friday.

Theorem

Recall

Let $p(x)$ be a polynomial and let c be a real number. Then $\lim_{x\to c} p(x) = p(c)$.

Theorem

Let $p(x)$ and $q(x)$ be polynomials, and let c be a real numbers. Then

$$\lim_{x\to c}\frac{p(x)}{q(x)} = \begin{cases} \dfrac{p(c)}{q(c)}, & \text{if } q(c)\neq 0 \\ \text{undefined}, & \text{if } p(c)\neq 0 \text{ and } q(c)=0 \end{cases}$$

If $p(c)=q(c)=0$ then more work is required to determine the limit.

"$\frac{0}{0}$" ind. form.

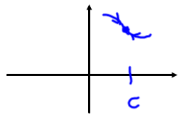
Note: We have learned that whenever $f(x)$ is a polynomial or rational function, and c is in the domain of f, then

$$\lim_{x\to c} f(x) = f(c)$$

i.e. we can evaluate the limit by evaluating the function.

Huge Property

What does this say about the graphs of polynomial and rational functions?

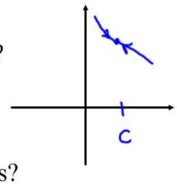


in the domain

This property is known as continuity.

What does it mean to say that $f(x)$ is continuous at $x = c$?

Geometrically?



In terms of limits?

def.

1. $f(c)$ exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Polynomials and Rational Functions are Continuous Everywhere They are Defined

From before

Terminology

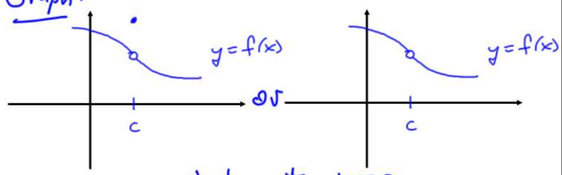
$f(x)$ is discontinuous at $x = c$ if and only if $f(x)$ is not continuous at $x = c$.

What are the Basic Types of Discontinuity?

1. Removable discontinuity.
2. Jump discontinuity.
3. Infinite discontinuity.

Removable Discontinuity at $x = c$.

Graph:



hole at $x=c$

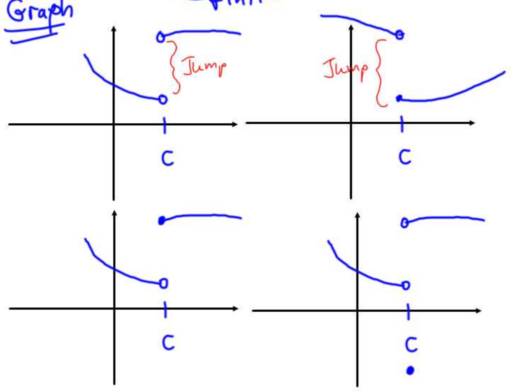
Def: $\lim_{x \to c} f(x)$ exists.

But $\lim_{x \to c} f(x) \neq f(c)$.

Jump Discontinuity at $x = c$.

Graph

Finite



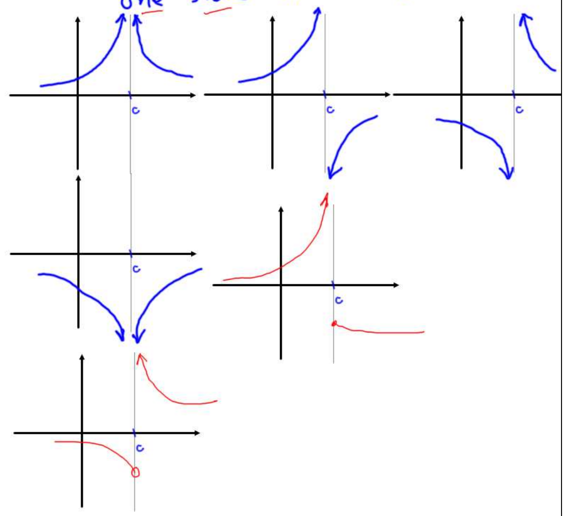
Def: $\lim_{x \to c^-} f(x)$ exists.

$\lim_{x \to c^+} f(x)$ exists.

But $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$.

Infinite Discontinuity at $x = c$

Vertical asymptote (at least from one side of $x=c$).



Example: Discuss the continuity of the following functions.

$$G(x) = 3x^3 - 2x^2 - 7$$

$$f(x) = \frac{x-1}{|x-1|}$$

$$H(x) = \frac{|x^2-4|}{x+2}$$

$$g(x) = \frac{x+2}{x^2+x-2}$$

Where is each function continuous? Also, classify any discontinuity.

$$G(x) = 3x^3 - 2x^2 - 7$$

Every real number is in the domain

Polynomial!!

$$\lim_{x \to c} G(x) = G(c)$$

at every domain value c.

$\therefore$ $G(x)$ is continuous at every value of x.

i.e. $G(x)$ is continuous on $(-\infty, \infty)$.

$$f(x) = \frac{x-1}{|x-1|}$$

We found: f is continuous at every value of x except $x = 1$. At $x = 1$, f has a jump discontinuity.

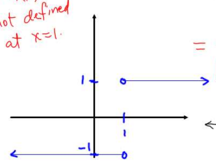
Note: $x=1$ is not in the domain.

Domain: $\underbrace{(-\infty, 1) \cup (1, \infty)}_{\text{every real \# except 1.}}$

Note: $|x-1| = \begin{cases} x-1, & \text{if } x \geq 1 \\ -(x-1), & \text{if } x < 1 \end{cases}$

$\therefore$ $f(x) = \frac{x-1}{|x-1|} = \begin{cases} \frac{x-1}{-(x-1)} & x < 1 \\ \frac{x-1}{x-1} & x > 1 \end{cases} = \begin{cases} -1, & x < 1 \; * \\ 1, & x > 1 \; * \end{cases}$

Note: $f(x)$ is not defined at $x=1$.



← we see a jump discont at $x=1$

In terms of limits:

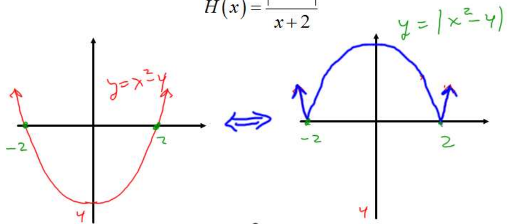
$*$ $\lim_{x \to 1^-} f(x) = -1$

$*$ $\lim_{x \to 1^+} f(x) = 1$

Both exists. But they are not equal.

Note: Since $f(x)$ is constant, both to the right and left of 1, $f(x)$ is continuous for every x value $\neq 1$.

$$H(x) = \frac{|x^2-4|}{x+2}$$



$$|x^2-4| = \begin{cases} x^2-4, & x \leq -2 \\ -(x^2-4), & -2 < x < 2 \\ x^2-4, & x \geq 2 \end{cases}$$

$\therefore$ $H(x) = \frac{|x^2-4|}{x+2} = \begin{cases} \frac{x^2-4}{x+2}, & x < -2 \\ \frac{-(x^2-4)}{x+2}, & -2 < x < 2 \\ \frac{x^2-4}{x+2}, & x \geq 2 \end{cases}$

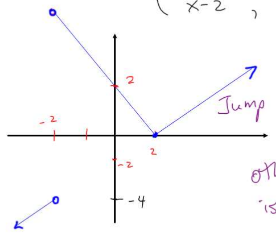
Note: $x=-2$ is not in the domain.

Domain: $(-\infty, -2) \cup (-2, \infty)$

Note: $\frac{x^2-4}{x+2} = \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} = x-2$ if $x \neq -2$.

$\therefore$ $H(x) = \begin{cases} x-2, & x < -2 \\ -(x-2), & -2 < x < 2 \\ x-2, & x \geq 2 \end{cases}$

$$\therefore \quad H(x) = \begin{cases} x-2, & x < -2 \\ -(x-2), & -2 < x < 2 \\ x-2, & x \geq 2 \end{cases}$$



$-(x-2) = -x+2$

Jump discont. at $x=-2$. otherwise, $H(x)$ is continuous.

We can show this using limits by first noting that the function breaks up into 3 lines. So the function is certainly continuous everywhere except possibly x=-2 and x=2. Let's check these.

$x=-2$:

$\lim_{x \to -2^-} H(x) = \lim_{x \to -2^-} (x-2) = -4$ ← Exist!

$\lim_{x \to -2^+} H(x) = \lim_{x \to -2^+} [-(x-2)] = 4$ ← But Not Equal.

$\therefore$ jump discont at $x=-2$.

$x=2$: $H(2)=0$

$\lim_{x \to 2^-} H(x) = \lim_{x \to 2^-} [-(x-2)] = 0$

$\lim_{x \to 2^+} H(x) = \lim_{x \to 2^+} (x-2) = 0$

$\Rightarrow \lim_{x \to 2} H(x) = 0$

$\therefore \lim_{x \to 2} H(x) = H(2)$

$\therefore$ $H(x)$ is continuous at $x=2$.

Putting all of this together, we find that $H(x)$ is continuous on

$(-\infty, -2) \cup (-2, \infty)$

AND the discontinuity at $x=-2$ is a jump discontinuity.

$$g(x) = \frac{x+2}{x^2+x-2}$$

↑ rational function.

Domain: All x values except those for which $\boxed{x^2+x-2=0}$.

$(x+2)(x-1) = 0$

$x=-2$ or $x=1$.

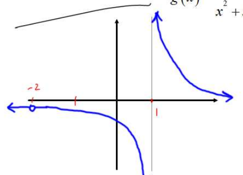
$\therefore$ Domain: $(-\infty,-2) \cup (-2,1) \cup (1,\infty)$.

AND $g(x) = \frac{x+2}{x^2+x-2}$ is a rational function. $\therefore$ $g(x)$ is continuous on $(-\infty,-2) \cup (-2,1) \cup (1,\infty)$.

Discontinuities occur at $x=-2$, $x=1$.

Let's Classify: $g(x) = \frac{x+2}{x^2+x-2} = \frac{x+2}{(x+2)(x-1)}$

$= \frac{1}{x-1}$, $x \neq -2$, $x \neq 1$



$\therefore$ Removable discont. at $x=-2$

Infinite discont. at $x=1$.

Give values for A and B so that

$$f(x) = \begin{cases} Ax-3 & x < -2 \\ 2 & x = -2 \\ x^2-B & x > -2 \end{cases}$$

is continuous.

← line; ← parabola; Piecewise

Note: The function is continuous both to the left of -2 and to the right of -2, because the function is defined in terms of polynomials on each side (regardless of values of A and B). So, we get the values of A and B by making sure the function is continuous at $x = -2$.

$f(-2) = 2$

$\lim_{x \to -2} f(x) = ?$

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} (Ax-3) = -2A-3$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (x^2-B) = 4-B$

$\therefore$ f is continuous at $x=-2$ if and only if

$-2A-3 = 4-B$ and $-2A-3 = 2$

$\Downarrow$

$A = -\frac{5}{2}$

$-2\left(-\frac{5}{2}\right)-3 = 4-B$

$2 = 4-B$

$B=2$

$\therefore$ f is continuous when $A=-\frac{5}{2}$ and $B=2$.