Math 1431

Jeff Morgan
jmorgan@math.uh.edu
651 PGH
Office Hours: 11:00-Noon MWF

No Office Hours Today

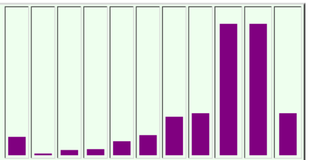
http://www.math.uh.edu/~jmorgan/Math1431
tinyurl.com/math1431
@morgancalculus

Next Monday is an Important Day!

- **Homework 2** due in lab/workshop.
- **EMCF05** is due online at 9:00am.
- **Online Quiz 1** is due at 11:59pm.
- **Poppers** start in lecture.
- **Access Codes** are due at 12:01am.

Purchase your Popper forms and Access Code from the UH Bookstore in the University Center.

Test 1 Grades



number of students	22	2	6	7	17	24	47	51	160	160	51
score range	0 to 10	10 to 20	20 to 30	30 to 40	40 to 50	50 to 60	60 to 70	70 to 80	80 to 90	90 to 100	100 to 110

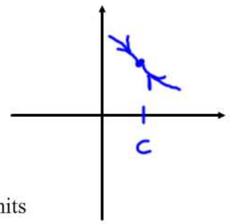
did not take the test

Median = 88

525 students took Test 1

Review of Continuity

Graphical Description

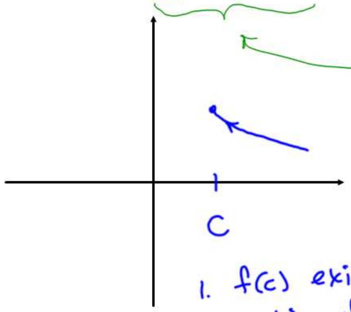


$f(x)$ is continuous at $x=c$

Definition in Terms of Limits

1. $f(c)$ exists
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

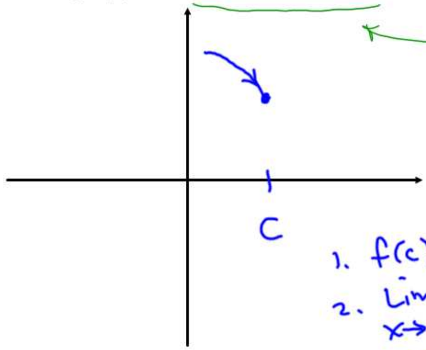
Question: What do you think it means to say that a function $f(x)$ is right continuous at $x = c$?



f is "continuous from the right"

1. $f(c)$ exists
2. $\lim_{x \to c^+} f(x)$ exists
3. $\lim_{x \to c^+} f(x) = f(c)$

Question: What do you think it means to say that a function $f(x)$ is left continuous at $x = c$?



continuous from the left

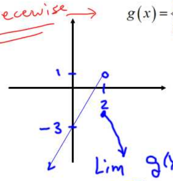
1. $f(c)$ exists
2. $\lim_{x \to c^-} f(x)$ exists
3. $\lim_{x \to c^-} f(x) = f(c)$

Example: Describe the continuity of the function below at $x = 2$.

piecewise $\rightarrow$ $g(x) = \begin{cases} 2x-3, & x < 2 \\ x-x^2, & x \geq 2 \end{cases}$

line

parabola



$g(2) = \boxed{-2}$

$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (2x-3) = 1$

These are not =. ∴ g is not left cont. at x=2

jump discont.

$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (x-x^2) = \boxed{-2}$

Since $g(2) = \lim_{x \to 2^+} g(x)$, g is right continuous at $x=2$.

Note: There is a jump discontinuity at $x=2$.

Question: What do you think it means to say that a function $f(x)$ is continuous on an interval?

① 

"closed interval a, b" $[a,b] = \{ x \mid a \leq x \leq b \}$

f is right continuous at a
f is left continuous at b
f is continuous at each $x=c$ with $a<c<b$.

② 

"half open, half closed interval a, b" $(a,b] = \{ x \mid a < x \leq b \}$

f is left continuous at b
f is continuous at each $x=c$ with $a<c<b$.

③

"half closed, half open a, b." interval

$[a,b) = \{x \mid a \le x < b\}$

f is right continuous at a

f is continuous at each $x=c$ with $a<c<b$.

④

"open interval a, b."

$(a,b) = \{x \mid a < x < b\}$

f is continuous at each $x=c$ with $a<c<b$.

An Important Fact Concerning Continuous Functions

Sums, differences, products, quotients and compositions of continuous functions are continuous on intervals on which they are defined.

Functions that are Continuous on their Domains of Definition

polynomials, rational functions, $|x|$, $\sqrt{x}$, x^r (with $r \neq 0$),

$\cos(x)$, $\sin(x)$, $\sec(x)$, $\csc(x)$, $\tan(x)$, $\cot(x)$

Example: Determine where $f(x) = \dfrac{\sqrt{x}-2}{x^2-4}$ is continuous, and describe any discontinuity.

f is created using differences and quotients of sqrt(x) and polynomials.

$\therefore$ f(x) will be continuous on intervals where it is define.

Domain: we can not have

$x^2-4=0$ ← no zero in denom.

$x<0$ ← no sqrt of negative #s

i.e. we cannot have

$x=-2,\ x=2$

$x<0$

$\therefore$ The domain is $[0,2)\cup(2,\infty)$.

$\therefore$ $f(x) = \dfrac{\sqrt{x}-2}{x^2-4}$ is continuous on $[0,2)\cup(2,\infty)$.

Q: What happens at $x=2$?

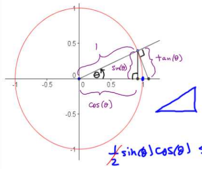
A: $\lim_{x\to 2} \dfrac{\sqrt{x}-2}{x^2-4} =$ dne

"$\frac{\sqrt{2}-2}{0}$"

b/c there is a vertical asympt at $x=2$.

$\therefore$ f(x) has an infinite discont at $x=2$.

Geometric Exploration:



$\cancel{\tfrac{1}{2}}\sin(\theta)\cos(\theta) \le \dfrac{\theta}{\cancel{2\pi}}\cdot\cancel{\pi} \le \cancel{\tfrac{1}{2}}\tan(\theta)$

$\cos(\theta) \le \dfrac{\theta}{\sin(\theta)} \le \dfrac{1}{\cos(\theta)}$

$\dfrac{1}{\cos(\theta)} \ge \dfrac{\sin(\theta)}{\theta} \ge \cos(\theta)$

let $\theta \to 0^+$

$1 \ge \lim_{\theta\to 0^+} \dfrac{\sin(\theta)}{\theta} \ge 1$

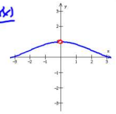
$\therefore \lim_{\theta\to 0^+} \dfrac{\sin(\theta)}{\theta} = 1$

More generally, $\lim_{\theta\to 0} \dfrac{\sin(\theta)}{\theta} = 1$

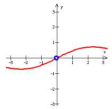
$\lim_{u\to 0} \dfrac{\sin(u)}{u} = 1$, $\boxed{\lim_{x\to 0} \dfrac{\sin(x)}{x} = 1}$

Two Important Limits

$y = \frac{\sin(x)}{x}$



$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1 \qquad ✱$$



$$\lim_{x\to 0}\frac{1-\cos(x)}{x} = 0$$

why? $\lim_{x\to 0} \frac{(1-\cos(x))}{x}\cdot\frac{(1+\cos(x))}{(1+\cos(x))}$

"$\frac{0}{0}$" ind. form

$$= \lim_{x\to 0}\frac{1-\cos^2(x)}{x\,(1+\cos(x))}$$

Recall $\lim_{x\to 0}\frac{\sin(x)}{x} = 1$ $\equiv$

$$= \lim_{x\to 0}\frac{\sin^2(x)}{x\,(1+\cos(x))}$$

$$= \lim_{x\to 0}\underbrace{\boxed{\frac{\sin(x)}{x}}}_{\to 1}\cdot\underbrace{\boxed{\frac{\sin(x)}{(1+\cos(x))}}}_{\to \frac{0}{2}=0}$$

$= 0$

Examples: $\lim_{x\to 0}\frac{\sin(2x)}{x} = \lim_{x\to 0} 2\cdot\boxed{\frac{\sin(2x)}{2x}} = 2\cdot 1 = 2$

"$\frac{0}{0}$" ind. form

use $\lim_{u\to 0}\frac{\sin(u)}{u} = 1$

$\lim_{u\to 0}\frac{u}{\sin(u)} = 1$

$$\lim_{x\to 0}\frac{\sin(2x)}{\sin(3x)} = \lim_{x\to 0} 2\cdot\underbrace{\boxed{\frac{\sin(2x)}{2x}}}_{\to 1}\cdot\underbrace{\boxed{\frac{3x}{\sin(3x)}}}_{\to 1}\cdot\frac{1}{3}$$

"$\frac{0}{0}$" ind. form

$= \frac{2}{3}$

$\lim_{x\to 0}\frac{x^2}{1-\cos^2(x)} =$ ← Next page

$\lim_{x\to 0}\frac{\tan(3x)}{2x^2-2x} =$ ← 2 pages forward

$$\lim_{x\to 0}\frac{x^2}{1-\cos^2(x)} = \lim_{x\to 0}\frac{x^2}{\sin^2(x)}$$

"$\frac{0}{0}$" ind. form

$$= \lim_{x\to 0}\underbrace{\boxed{\frac{x}{\sin(x)}}}_{\to 1}\cdot\underbrace{\boxed{\frac{x}{\sin(x)}}}_{\to 1}$$

Recall $\lim_{u\to 0}\frac{u}{\sin(u)} = 1$

$= 1$

$$\lim_{x\to 0}\frac{\tan(3x)}{2x^2-2x} = \lim_{x\to 0}\frac{\sin(3x)\cdot\frac{1}{\cos(3x)}}{x\cdot(2x-2)}$$

"$\frac{0}{0}$" ind. form

$$= \lim_{x\to 0} 3\cdot\underbrace{\boxed{\frac{\sin(3x)}{3x}}}_{\to 1}\cdot\underbrace{\boxed{\frac{1}{\cos(3x)(2x-2)}}}_{\to \frac{1}{-2}}$$

$\lim_{u\to 0}\frac{\sin(u)}{u} = 1$

$= -\frac{3}{2}$

Warning: $\lim_{x\to 1}\frac{\sin(x)}{x} = \frac{\sin(1)}{1}$ $\equiv$

Pay attention!