Math 1431

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Office Hours: 11-noon MWF

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Online Quizzes are available.

Homework 3 is due next Monday in lab/workshop.

EMCF07 is due Friday morning at 9:00am.

There is a Written Quiz in lab/workshop on Friday.

Alpha Lambda Delta Honor Society Movie Night—Free to everyone!!
6pm in Cougar Village Rm. N111. Snacks will be provided!

An Introduction to Derivatives: How can we approximate the slope of the tangent line to the graph at \( x = a \)?

“Slope of the graph at \( x = a \).”

\[
\frac{f(a+h) - f(a)}{h}
\]

\( (a, f(a)) \) and \( (a+h, f(a+h)) \)
We can approximate the tangent line to the graph at $x = a$ by using a secant line.

\[ y = f(x) \]

\[ (a, f(a)) \quad (a + h, f(a + h)) \]

II.

The approximation will continue to improve as we make $h$ even smaller.

\[ \text{slope of the secant line} = \frac{f(a + h) - f(a)}{h} \]

IV.

The slope of the tangent line at $x = a$ is called the derivative of $f$ at $x = a$ and it is denoted by $f'(a)$.

V.

If the limit exists, then we can find the slope of the graph at $x = a$ by taking a limit.

\[ \text{slope of the tangent line at } x = a \text{ is } \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \]

\[ f'(a) \]

"f prime of a"
Example: Find the derivative of \( f(x) = x^2 \) at \( x = 1 \), and give the equation of the tangent line to the graph of \( f \) at the point where \( x = 1 \).

\[
\begin{align*}
\frac{\text{d}f}{\text{d}x} (1) & = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} \\
& = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h} \\
& = \lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h} \\
& = \lim_{h \to 0} \frac{2h + h^2}{h} \\
& = \lim_{h \to 0} (2 + h) = 2 \\
\therefore \frac{\text{d}f}{\text{d}x} (1) & = 2.
\end{align*}
\]

\( \frac{\text{d}f}{\text{d}x} (1) = 2 \).

Example: Find the derivative of \( f(x) = x^2 \).

\[
\begin{align*}
\frac{\text{d}f}{\text{d}x} (x) & = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
& = \lim_{h \to 0} \frac{((x+h)^2 - x^2)}{h} \\
& = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\
& = \lim_{h \to 0} \frac{2xh + h^2}{h} \\
& = \lim_{h \to 0} (2x + h) = 2x \\
\therefore \frac{\text{d}f}{\text{d}x} (x) & = 2x.
\end{align*}
\]

Popper P02

1. \( f(x) = x^2 \). Give \( f'(2) \).

2. \( f(x) = x^2 \). Give the slope of the tangent line to the graph of \( f(x) \) at \( x = -1 \).
The Derivative: Overview...

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{if } f \text{ is differentiable} \]

Other notation: \( \frac{d}{dx} f(x) = f'(x) \)

Note: \( f'(x) \) is the slope of the tangent line at \( x = a \).

Interpretations of the Derivative

<table>
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<th>Function</th>
<th>Description</th>
<th>Derivative</th>
<th>Interpretation</th>
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<tbody>
<tr>
<td>( F(x) )</td>
<td>Standard function.</td>
<td>( F'(x) )</td>
<td>Slope function.</td>
</tr>
<tr>
<td>( F(x) )</td>
<td>Standard function.</td>
<td>( F'(x) )</td>
<td>Instantaneous rate of change of ( F(x) ) with respect to ( x ).</td>
</tr>
<tr>
<td>( s(t) )</td>
<td>Position at time ( t ).</td>
<td>( s'(t) )</td>
<td>Velocity, sometimes named ( v(t) ).</td>
</tr>
<tr>
<td>( v(t) )</td>
<td>Velocity at time ( t ).</td>
<td>( v'(t) )</td>
<td>Acceleration, sometimes named ( a(t) ).</td>
</tr>
</tbody>
</table>

Notation: \( f(x) \) is differentiable at \( x = a \) if and only if \( f'(a) \) exists.

(i.e. if and only if \( f \) has a derivative at \( x = a \))

Example: Give a formula for the derivative of \( f(x) = \frac{1}{x+1} \).

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ = \lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \]

\[ = \lim_{h \to 0} \frac{\frac{x+1 - (x+h+1)}{(x+h+1)(x+1)}}{h} \]

\[ = \lim_{h \to 0} \frac{-1}{(x+1)(x+h+1)(x+1)} \]

\[ = -\frac{1}{(x+1)^2} \]

\[ \therefore f'(x) = -\frac{1}{(x+1)^2} \]

Popper P02

\[ f(x) = \frac{1}{x+1} \]

3. \( f(x) \) is the function in the previous example. Give \( f'(1) \).

4. \( f(x) \) is the function in the previous example. Give the slope of the tangent line to the graph of \( f(x) \) at \( x = -2 \).
Example: Give an equation for the tangent line to the graph of \( f(x) = \frac{1}{x+1} \) at \( x = 2 \).

\[
\begin{align*}
\frac{df}{dx} &= \frac{-1}{(x+1)^2} \\
\text{Slope} &= f'(2) = -\frac{1}{9} \\
\text{Point} &= (2, f(2)) = (2, \frac{1}{3}) \\
\text{Eq. for T.L. at } x = 2 : & \\
& y - \frac{1}{3} = -\frac{1}{9}(x-2)
\end{align*}
\]

How can we use the derivative to find the slope of the normal line to the graph of \( f(x) \) at \( x = a \)?

Normal line is \( \perp \) to the tangent line, and it passes through \((a, f(a))\).

\[
\text{Slope of N.L.} = -\frac{1}{\frac{df}{dx}(a)}
\]

provided \( \frac{df}{dx}(a) \neq 0 \).

Example: Give an equation for the normal line to the graph of \( f(x) = \frac{1}{x+1} \) at \( x = 1 \).

\[
\begin{align*}
\frac{df}{dx} &= \frac{-1}{(x+1)^2} \Rightarrow f'(1) = -\frac{1}{4} \\
\text{Slope of N.L.} &= -\frac{1}{f'(1)} = 4 \\
\text{Point} &= (1, f(1)) = (1, \frac{1}{2}) \\
\text{Eq. for N.L. : } & \\
& y - \frac{1}{2} = 4(x-1)
\end{align*}
\]

Popper P02

5. \( f(x) \) is the function in the previous example. Give the slope of the normal line to the graph of \( f(x) \) at \( x = -2 \).
How is the derivative related to continuity?

See the video!!

How can the graph of a function be used to determine where a function is not differentiable?

See the video!!

The function $f(x)$ is graphed below. Determine the values of $x$ where $f$ is differentiable.

See the video!!