Math 1431

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tinyurl.com/math1431
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Online Quizzes are available.
→ #2

Homework 3 is due next Monday in lab/workshop.

EMCF07 is due Friday morning at 9:00am.

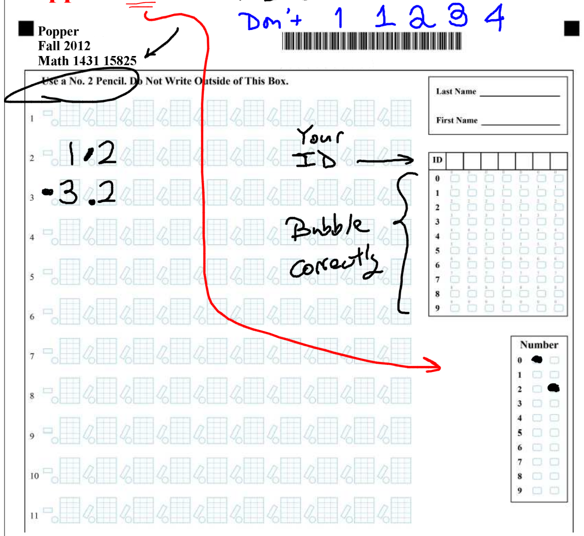
There is a **Written Quiz** in lab/workshop on Friday.

Alpha Lambda Delta Honor Society Movie Night – Free to everyone!!
6pm in Cougar Village Rm. N111. **Snacks will be provided!**

Tomorrow

Popper P02

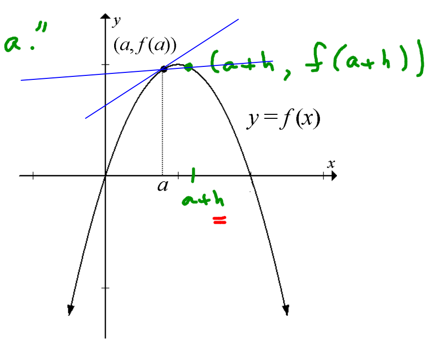
1 2 3 4 5 6 7 8 9 0
Don't 1 1 2 3 4



An Introduction to Derivatives: How can we approximate the slope of the tangent line to the graph at $x = a$?

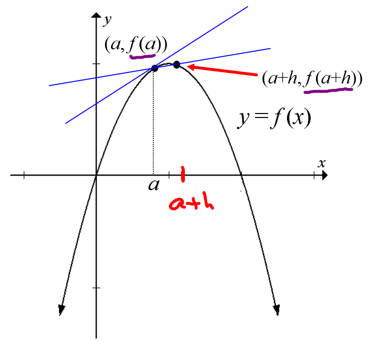
"slope of the graph at $x = a$."

I.



We can approximate the tangent line to the graph at $x = a$ by using a secant line.

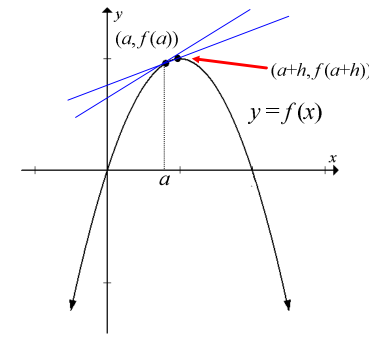
II.



slope of the secant line $= \dfrac{f(a+h) - f(a)}{h} = \dfrac{\Delta y}{\Delta x}$

We can improve this approximation by making h smaller.

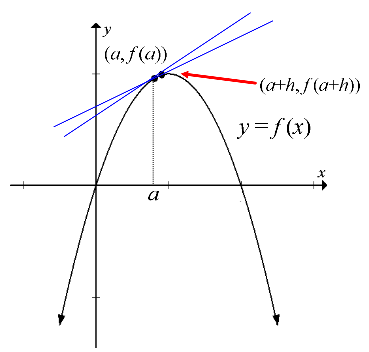
III.



slope of the secant line $= \dfrac{f(a+h) - f(a)}{h}$

The approximation will continue to improve as we make h even smaller.

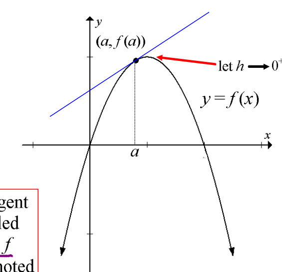
IV.



slope of the secant line $= \dfrac{f(a+h) - f(a)}{h}$

If the limit exists, then we can find the slope of the graph at $x = a$ by taking a limit.

V.



The slope of the tangent line at $x = a$ is called **the derivative** of f at $x = a$, and it is denoted by $f\,'(a)$

slope of the tangent line at $x=a$ $= \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$=$

$f'(a)$

"f prime of a"

Example: Find the derivative of $f(x)=x^2$ at $x=1$, and give the equation of the tangent line to the graph of f at the point where $x=1$.

① $$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{(1+h)^2-1}{h}$$

"$\frac{0}{0}$" ind form

$$= \lim_{h\to 0}\frac{\cancel{1}+2h+h^2-\cancel{1}}{h}$$

$$= \lim_{h\to 0}\frac{2\cancel{h}+h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h\to 0}(2+h) = 2$$

$\therefore$ $f'(1)=2$.

② To get the T.L. at $x=1$:

$f(x)=x^2$

slope: $f'(1) = 2$

Point: $(1, f(1)) = (1,1)$

Equation: $y-1 = 2(x-1)$

or

$y = 2x-1$

Example: Find the derivative of $f(x)=x^2$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

function of x

"slope function"

$$= \lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

"$\frac{0}{0}$"

$$= \lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$= \lim_{h\to 0}\frac{2x\cancel{h}+h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h\to 0}(2x+h) = 2x$$

$\therefore$ $f(x)=x^2$

$\Rightarrow$ $f'(x)=2x$

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1. $f(x)=x^2$. Give $f'(2)$.

2. $f(x)=x^2$. Give the slope of the tangent line to the graph of $f(x)$ at $x=-1$.

The Derivative: Overview...

$$\underline{f'(x)} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ if it exists

function

Other notation: $\frac{d}{dx} f(x) = f'(x)$ // $\frac{df(x)}{dx}$ ⇒ $f(x)$ x

Note: $f'(a)$ is the slope of the tangent line at $x = a$.

Interpretations of the Derivative

Function	Description	Derivative	Interpretation
$F(x)$	Standard function.	$F'(x)$	Slope function.
$F(x)$	Standard function.	$F'(x)$	Instantaneous rate of change of $F(x)$ with respect to x.
$s(t)$	Position at time t.	$s'(t)$	Velocity, sometimes named $v(t)$.
$v(t)$	Velocity at time t.	$v'(t)$	Acceleration, sometimes named $a(t)$.

Notation: $f(x)$ is differentiable at $x = a$ if and only if $f'(a)$ exists.
(i.e. if and only if f has a derivative at $x = a$)

Example: Give a formula for the derivative of $f(x) = \frac{1}{x+1}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

"$\frac{0}{0}$"

$$= \lim_{h \to 0} \frac{\frac{x+1-(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \frac{-1}{(x+1)^2}$$

$$\therefore\ f(x) = \frac{1}{x+1} \Rightarrow f'(x) = \frac{-1}{(x+1)^2}$$

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$f(x) = \frac{1}{x+1}$, $f'(x) = \frac{-1}{(x+1)^2}$

3. $f(x)$ is the function in the previous example. Give $f'(1)$.

4. $f(x)$ is the function in the previous example. Give the slope of the tangent line to the graph of $f(x)$ at $x = -2$.

Example: Give an equation for the tangent line to the graph of $f(x)=\dfrac{1}{x+1}$ at $x=2$.

$$f'(x)=\frac{-1}{(x+1)^2}$$

slope $= f'(2) = -\frac{1}{9}$

Point $= (2, f(2)) = (2, \frac{1}{3})$

Eq. for T.L. at $x=2$:

$$y-\frac{1}{3}=-\frac{1}{9}(x-2)$$

How can we use the derivative to find the slope of the normal line to the graph of $f(x)$ at $x=a$?

Normal line is $\perp$ to the tangent line, and it passes through $(a, f(a))$.

Slope of N.L. $= \dfrac{-1}{f'(a)}$

provided $f'(a)\neq 0$.

Example: Give an equation for the normal line to the graph of $f(x)=\dfrac{1}{x+1}$ at $x=1$.

$$f'(x)=\frac{-1}{(x+1)^2} \Rightarrow f'(1)=-\frac{1}{4}$$

slope of N.L. $= \dfrac{-1}{f'(1)} = 4$

point $= (1, f(1)) = (1, \frac{1}{2})$

Equation for N.L.: $y-\frac{1}{2}=4(x-1)$

Popper P02

5. $f(x)$ is the function in the previous example. Give the slope of the normal line to the graph of $f(x)$ at $x=-2$.

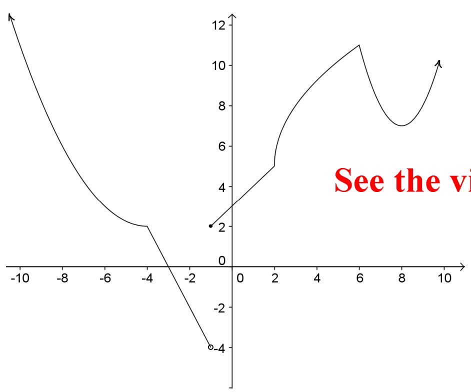
How is the derivative related to continuity?

See the video!!

How can the graph of a function be used to determine where a function is not differentiable?

See the video!!

The function $f(x)$ is graphed below. Determine the values of x where f is differentiable.



See the video!!