There is a **Written Quiz** in lab today.

**Online Quiz 2** is due on Monday at 11:59pm.

An **EMCF** is due on Monday.

**Homework 3** is due on Monday.

**Help videos** are posted for Sections 3.1 and 3.2.

We are in Chapter 3!

www.math.uh.edu/~jmorgan/Math1431

tinyurl.com/math1431
@morgancalculus

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**The Derivative: Overview**...

\[
\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

**prime of f**

Note: \(f'(x)\) is the slope of the tangent line at \(x = a\).

\[
\frac{d}{dx}f(x) = f'(x)
\]

(provided \(f'(a) \neq 0\))

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**Popper P03**

Do not write 1 or 1 or 4

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How is the derivative related to continuity?

If \( f(x) \) is differentiable at \( x = a \), then \( f(x) \) is continuous at \( x = a \).

But, the converse is not true.

\( f(x) = |x| \)

is continuous.

\( f(x) = |x| \) does not have a derivative at \( x = 0 \).

How can the graph of a function be used to determine where a function is not differentiable?

A function is not differentiable at:

1. Points of discontinuity.
2. Points where the graph has a corner.
3. Points where the graph has a cusp.
4. Points where the graph has a vertical tangent.

\[ f(x) = x^\frac{1}{3} \]

\[ \text{at } x = 0 \]

\[ f(x) = x^\frac{1}{3} \]

\[ \text{at } x = 0 \]

The function \( f(x) \) is graphed below. Determine the values of \( x \) where \( f \) is differentiable.

\[ f \text{ is differentiable on } (\infty, -4) \cup (-4, -1) \cup (-1, 0) \cup (0, 2) \cup (2, \infty) \]

An Exercise from 3.1

21. The graph of a function \( f \) is shown in the figure.

(a) For which numbers \( c \) does \( f \) fail to be continuous? For each discontinuity, state whether it is a removable discontinuity, a jump discontinuity, or neither.

(b) At which numbers \( c \) is \( f \) continuous but not differentiable?

\[ x = -1, \text{ removable} \]

\[ x = 1, \text{ jump} \]

\[ x = 0 \]

\[ x = 3 \]
Identify the values of $x$ where the slope of the tangent line is **positive**.

$(-\infty, a) \cup (c, \infty)$

$y = f(x)$

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Identify the values of $x$ where the slope of the tangent line is **negative**.

$\left(a, c\right)$

$\alpha < x < \beta$

$y = f(x)$

---

Identify the values of $x$ where the slope of the tangent line is **zero**.

$x = a$ and $x = c$

$y = f(x)$

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**Algebraic Properties of the Derivative**

1. Derivatives of sums, differences, scalar multiples, products and quotients.

2. Power rule.
Sums, Differences and Scalar Multiples

If \( f \) and \( g \) are differentiable and \( c \) is a scalar, then \( f + g \), \( fg \) and \( cf \) are differentiable. Furthermore,

1. \[ \frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x) \]
2. \[ \frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x) \]
3. \[ \frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}f(x) \]

Examples:

Give the derivative of \( f(x) = x^2 \).

\[
\begin{align*}
 f'(x) &= \frac{d}{dx} x^2 = \frac{d}{dx} (x \cdot x) \\
 &= x \cdot \frac{d}{dx} x + x \cdot \frac{d}{dx} x \\
 &= x \cdot 1 + x \cdot 1 \\
 &= 2x
\end{align*}
\]

Give the derivative of \( R(x) = x^3 + x \).

\[
\begin{align*}
 R'(x) &= \frac{d}{dx} (x^3 + x) = \frac{d}{dx} x^3 + \frac{d}{dx} x \\
 &= \frac{d}{dx} (x^3 \cdot x) + 1 \\
 &= x^3 \frac{d}{dx} x + x \cdot \frac{d}{dx} x^2 + 1 \\
 &= x^3 \cdot 1 + x \cdot 2x + 1 \\
 &= 3x^2 + 1
\end{align*}
\]

Products and Quotients

If \( f \) and \( g \) are differentiable then \( f \cdot g \) and \( \frac{f}{g} \) are differentiable. Furthermore,

4. \[ \frac{d}{dx}(f(x) \cdot g(x)) = f(x) \frac{d}{dx}g(x) + g(x) \frac{d}{dx}f(x) \]
5. \[ \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = g(x) f'(x) - f(x) g'(x) \]
\[ g(x)^2 \]

Quotient rule

Popper P03

1. We found the derivative of \( f(x) = x^2 \) on the previous page. Give the slope of the tangent line to the graph of \( f \) at the point where \( x = -2 \).

2. We found the derivative of \( f(x) = x^2 \) on the previous page. Give the slope of the normal line to the graph of \( f \) at the point where \( x = -2 \).

3. We found the derivative of \( R(x) = x^3 + x \) on the previous page. Give the value of \( R'(1) \).
**Power Rule**

\[
\frac{d}{dx} x^n = nx^{n-1}, \quad n \neq 0
\]

\[
\begin{align*}
\frac{d}{dx} x^2 &= 2x \\
\frac{d}{dx} x^3 &= 3x^2 \\
\frac{d}{dx} x^4 &= 4x^3 \\
\frac{d}{dx} x^5 &= 5x^4
\end{align*}
\]

**Examples:**

Give the derivative of \( f(x) = x^4 + 2x^3 \).

\[
f'(x) = \frac{d}{dx} (x^4 + 2x^3) = \frac{d}{dx} x^4 + \frac{d}{dx} 2x^3
\]

\[
= 4x^3 + 2 \cdot \frac{d}{dx} x^3 = 4x^3 + 2 \cdot 3x^2 = 4x^3 + 6x^2
\]

Give the derivative of \( g(x) = (\sqrt{x} + 1)(3x^2 - 2x + 2) \).

\[
g'(x) = \frac{d}{dx} (u(x)v(x)) = u(x)v'(x) + v(x)u'(x)
\]

\[
= (\sqrt{x} + 1) \left[ 3 \cdot 3x - 2 \right] + (3x^2 - 2x + 2) \frac{d}{dx} (\sqrt{x} + 1)
\]

\[
= (\sqrt{x} + 1) \left[ 3 \cdot 3x - 2 \right] + (3x^2 - 2x + 2) \left( \frac{1}{2\sqrt{x}} \right)
\]

Example:

Give the derivative of \( H(x) = 3x^3 - 5x^2 - 6x + 3 \).

\[
H'(x) = 9x^2 - 10x - 6
\]

Example:

Give the derivative of \( g(x) = \frac{2}{\sqrt{x}} - \frac{3}{x} + x^2 - 1 \).

\[
g'(x) = 2 \cdot (-\frac{1}{2}) x^{-\frac{3}{2}} - 3 \cdot (-1) x^{-2} + 2x - 0
\]

\[
g'(x) = x^{-\frac{3}{2}} + 3x^{-2} + 2x - 0
\]
Example: Give the derivative of \( f(x) = \frac{2x-3}{x^2+1} \).

See the video