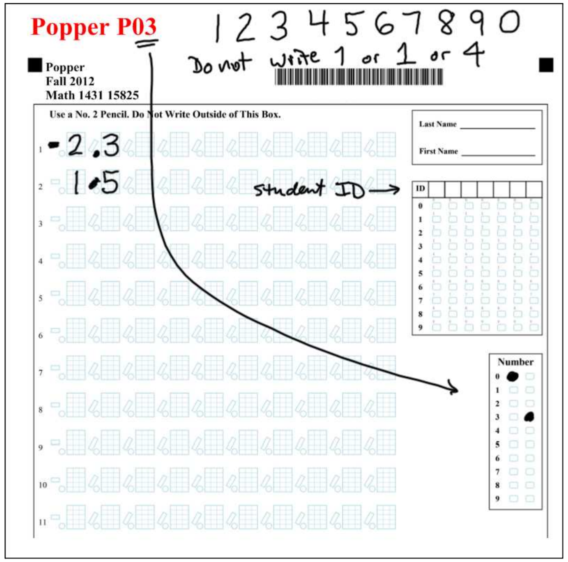
Math 1431
Jeff Morgan

- There is a **Written Quiz** in lab today.
- **Online Quiz 2** is due on Monday at 11:59pm.
- An **EMCF** is due on Monday.
- **Homework 3** is due on Monday.
- **Help videos** are posted for Sections 3.1 and 3.2.
- We are in Chapter 3!

www.math.uh.edu/~jmorgan/Math1431
tinyurl.com/math1431
@morgancalculus



The Derivative: Overview...

Instantaneous rate of change

slope function

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ if the limit exists

"f prime of x"

$D_x f(x)$

Other notation: $\frac{d}{dx} f(x) = f'(x)$

$\frac{df(x)}{dx}$

Note: $f'(a)$ is the slope of the tangent line at $x = a$.

$-\frac{1}{f'(a)}$ is the slope of the normal line at $x=a$

(provided $f'(a) \neq 0$)

How is the derivative related to continuity?

If $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.

However, the converse is not true.

ex. $f(x) = |x|$ at $x=0$.

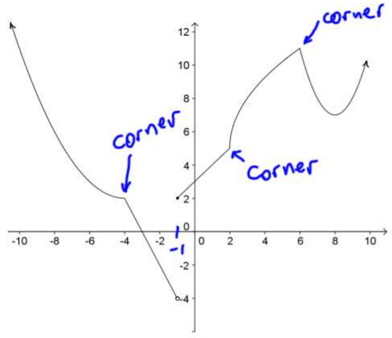
How can the graph of a function be used to determine where a function is not differentiable?

A function is not differentiable at:

1. Points of discontinuity.
2. Points where the graph has a corner.
3. Points where the graph has a cusp. ← $f(x) = x^{2/3}$ at $x = 0$
4. Points where the graph has a vertical tangent.

$f(x) = x^{1/3}$ at $x = 0$

The function $f(x)$ is graphed below. Determine the values of x where f is differentiable.



f is not differentiable at

$x = -4$,

$x = -1$,

$x = 2$,

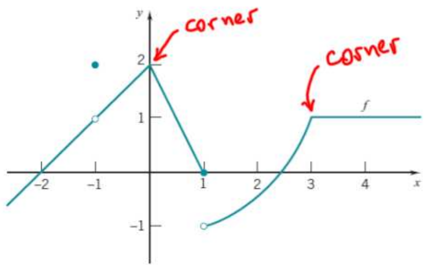
$x = 6$

$\therefore$ f is differentiable on

$(-\infty, -4) \cup (-4, -1) \cup (-1, 2) \cup (2, 6) \cup (6, \infty)$

An Exercise from 3.1

21. The graph of a function f is shown in the figure.



(a) For which numbers c does f fail to be continuous? For each discontinuity, state whether it is a removable discontinuity, a jump discontinuity, or neither.

$x = -1$, removable

$x = 1$, jump

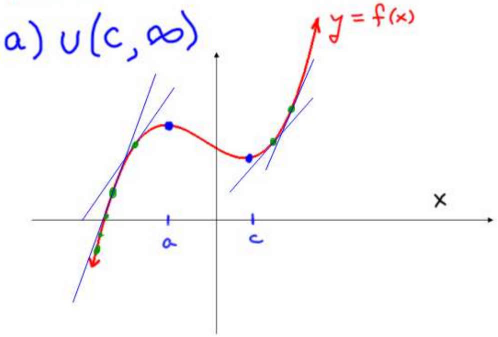
(b) At which numbers c is f continuous but not differentiable?

$x = 0$ and

$x = 3$

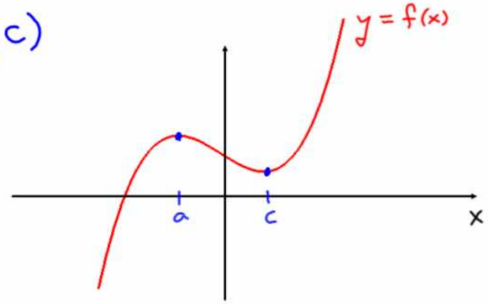
Identify the values of x where the slope of the tangent line is **positive**.

$(-\infty, a) \cup (c, \infty)$



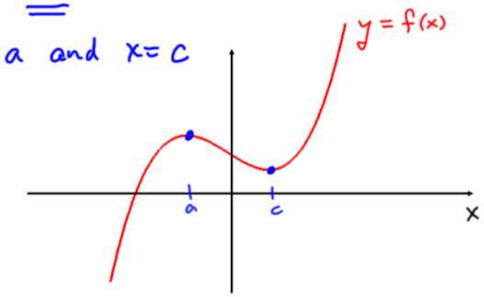
Identify the values of x where the slope of the tangent line is **negative**.

(a, c)



Identify the values of x where the slope of the tangent line is **zero**.

$x = a$ and $x = c$



Algebraic Properties of the Derivative

Formulas

Sums, Differences and Scalar Multiples

If f and g are differentiable and c is a scalar, then $f+g$, f-g and cf are differentiable. Furthermore,

1. $\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
2. $\frac{d}{dx}(f(x)-g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$
3. $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$

Why? Set $F(x) = f(x) + g(x)$.

Then $F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$

$$= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h\to 0} \left[\underbrace{\frac{f(x+h) - f(x)}{h}}_{f'(x)} + \underbrace{\frac{g(x+h) - g(x)}{h}}_{g'(x)} \right]$$

$$= f'(x) + g'(x)$$

Products and Quotients

If f and g are differentiable then $f \cdot g$ and f/g are differentiable. Furthermore,

4. $\frac{d}{dx}(f(x)\cdot g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$ ← product rule

5. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ ← quotient rule

Examples:

Give the derivative of $f(x) = x^2$. $\Rightarrow f'(x) = 2x$

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(x\cdot x) = x\frac{d}{dx}x + x\cdot\frac{d}{dx}x$$
$$= x\cdot 1 + x\cdot 1$$
$$= 2x$$

Give the derivative of $R(x) = x^3 + x$.

$$R'(x) = \frac{d}{dx}(x^3 + x) = \frac{d}{dx}x^3 + \frac{d}{dx}x$$
$$= \frac{d}{dx}(x^2\cdot x) + 1$$
$$= x^2\cdot\frac{d}{dx}x + x\cdot\frac{d}{dx}x^2 + 1$$
$$= x^2\cdot 1 + x\cdot 2x + 1$$
$$\therefore \quad R'(x) = 3x^2 + 1$$

Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}, \quad n \neq 0$$

$$\frac{d}{dx}x^3 = 3x^2$$
$$\frac{d}{dx}x^4 = 4x^3$$
$$\frac{d}{dx}x^5 = 5x^4$$

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-\frac{1}{2}}$$
$$= \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\frac{1}{x^2} = \frac{d}{dx}x^{-2}$$
$$= -2x^{-3}$$

Examples:

Give the derivative of $f(x) = x^4 + 2x^3$.

$$f'(x) = \frac{d}{dx}(x^4 + 2x^3)$$
$$= \frac{d}{dx}x^4 + \frac{d}{dx}2x^3 = 4x^3 + 2\cdot 3x^2$$
$$= 4x^3 + 6x^2$$

Give the derivative of $g(x) = \underbrace{\underbrace{(\sqrt{x}+1)}_{u(x)}\underbrace{(3x^3 - 2x + 2)}_{v(x)}}_{\text{product}}$.

Recall: $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2}$

Recall: $\frac{d}{dx}(u(x)\cdot v(x)) = u(x)\cdot v'(x) + v(x)\cdot u'(x)$

$$g'(x) = (\sqrt{x}+1)\cdot\frac{d}{dx}(3x^3 - 2x + 2) + (3x^3 - 2x + 2)\frac{d}{dx}(\sqrt{x} + 1)$$
$$= (\sqrt{x}+1)\cdot(3\cdot 3x^2 - 2 + 0) + (3x^3 - 2x + 2)\left(\tfrac{1}{2}x^{-\frac{1}{2}} + 0\right)$$
$$= (\sqrt{x}+1)(9x^2 - 2) + (3x^3 - 2x + 2)\cdot\frac{1}{2\sqrt{x}}$$

Example:

Give the derivative of $H(x)=3x^3-5x^2-6x+3$.

$$H'(x)=\frac{d}{dx}\left(3x^3-5x^2-6x+3\right)$$

$$=3\frac{d}{dx}x^3-5\frac{d}{dx}x^2-6\frac{d}{dx}x+\frac{d}{dx}3$$

$$=3\cdot 3x^2-5\cdot 2x-6+0$$

$$=9x^2-10x-6$$

Example:

Give the derivative of $g(x)=\frac{2}{\sqrt{x}}-\frac{3}{x}+x^2-1$.

$$g'(x)=\frac{d}{dx}\left(\frac{2}{\sqrt{x}}-\frac{3}{x}+x^2-1\right)$$

$$=\frac{d}{dx}\left(2x^{-\frac{1}{2}}-3x^{-1}+x^2-1\right)$$

$$=2\frac{d}{dx}x^{-\frac{1}{2}}-3\frac{d}{dx}x^{-1}+\frac{d}{dx}x^2-\frac{d}{dx}1$$

$$=2\cdot\left(-\tfrac{1}{2}\right)x^{-\frac{3}{2}}-3\cdot(-1)x^{-2}+2x-0$$

$$=-x^{-3/2}+3x^{-2}+2x$$

Example: Give the derivative of $f(x)=\boxed{\frac{2x-3}{x^2+1}}$. $\leftarrow u(x)$ $\leftarrow v(x)$

quotient

Recall:

$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right)=\frac{v(x)u'(x)-u(x)v'(x)}{v(x)^2}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{2x-3}{x^2+1}\right)=\frac{(x^2+1)\cdot\frac{d}{dx}(2x-3)-(2x-3)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$=\frac{(x^2+1)\cdot 2-(2x-3)(2x+0)}{(x^2+1)^2}$$

$$=\frac{2x^2+2-4x^2+6x}{(x^2+1)^2}$$

$$=\frac{-2x^2+6x+2}{(x^2+1)^2}$$