Notes:

- This morning - EMCF09
- Friday - EMCF10, Written Quiz
- Monday - EMCF11, Homework, Quiz 3
- October 4, 5, 6 - Test 2 (in CASA). The scheduler will open on Sept. 20th at 12:01am.

We will have class on the days it is scheduled.

Recall

$$\frac{d}{dx}\sin(x)=\cos(x) \qquad \frac{d}{dx}\cos(x)=-\sin(x)$$

Consequences

$$\frac{d}{dx}\tan(x)=\sec^2(x) \qquad \frac{d}{dx}\cot(x)=-\csc^2(x)$$

$$\frac{d}{dx}\sec(x)=\sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x)=-\csc(x)\cot(x)$$

$$\frac{d}{dx}\frac{\sin(x)}{\cos(x)}=\frac{\cos(x)\cos(x)-\sin(x)\cdot(-\sin(x))}{\cos^2(x)}$$

$$=\frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}=\frac{1}{\cos^2(x)}=\sec^2(x)$$

Example: Give the derivative of $f(x)=\sin(x)-3\tan(x)$.

$$f'(x)=\cos(x)-3\sec^2(x)$$

?

Question: What does Geogebra give for this derivative? Is it correct?

- $f'(x)=\cos(x)-3\tan^2(x)-3$

$$=\cos(x)-3\left(\tan^2(x)+1\right)$$

$\sec^2(x)$

Yes

Note: If you write your answers in decimal form on a Popper, then make sure the answer is accurate to 4 decimal places.

Example: Put your calculator in radian mode. cos(2) = -0.4161468365...
You can enter this on the Popper form as -0.4161.

You will not be able to use a calculator on the proctored Tests.

If your answer is 2.112193263 then input 2.1121

Popper P05

1. $f(x) = 2\sqrt{x} + \cos(x)$. $f'(2) =$

2. $f(x) = 2\sqrt{x} + \cos(x)$. Give the y-intercept of the tangent line to the graph at $x = 2$.

DO NOT USE the trancated answer from #1 to get the answer to #2. This can cause errors.

Example: Give the derivative of $g(x) = \dfrac{\sin(x)}{x + \cot(x)}$.

quotient

$$g'(x) = \frac{(x+\cot(x))\cos(x) - \sin(x)(1 - \csc^2(x))}{(x+\cot(x))^2}$$

Using Wolfram Alpha for Homework Help

www.wolframalpha.com

See my comments in the video.

Examples:

The Chain Rule

If u is differentiable at x and f is differentiable at $u(x)$, then the composition $f \circ u$ is differentiable at x and $(f \circ u)'(x) = f'(u(x))\,u'(x)$.

y u'(x)

$$\text{i.e. } \frac{d}{dx} f\big(u(x)\big) = f'\big(u(x)\big)\frac{d}{dx}u(x)$$

$\frac{df(u(x))}{dx}$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Why?

$g(x) = f(u(x))$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(u(x+h))-f(u(x))}{h}$$

"$\frac{0}{0}$"

$$= \lim_{h\to 0} \frac{f(u(x+h))-f(u(x))}{u(x+h)-u(x)} \cdot \frac{u(x+h)-u(x)}{h}$$

$u(x)+k$; k ; $u'(x)$

as $h\to 0$, $k\to 0$

$$= \lim_{k\to 0} \frac{f(u(x)+k)-f(u(x))}{k} \cdot u'(x)$$

$$= f'(u(x))\cdot u'(x)$$

Examples:

$f(x)$	$f'(x)$
$\cos(2x)$	$-\sin(2x)\cdot 2 = -2\sin(2x)$
$\sin(3x)$	$\cos(3x)\cdot 3 = 3\cos(3x)$
$(x^2+2)^4$	$4(x^2+2)^3\cdot 2x$
$\left(2x-\frac{3}{x}\right)^5$	$5\left(2x-\frac{3}{x}\right)^4\cdot\left(2+\frac{3}{x^2}\right)$
$x\sin(\pi x)$	$x\cdot\frac{d}{dx}\sin(\pi x)+\sin(\pi x)$ $= x\cos(\pi x)\pi + \sin(\pi x)$ $= \pi x\cos(\pi x) + \sin(\pi x)$

$\frac{d}{dx}\cos(u(x)) = -\sin(u(x))\cdot u'(x)$

$\frac{d}{dx}\sin(u(x)) = \cos(u(x))\cdot u'(x)$

$\frac{d}{dx}(u(x))^4 = 4u(x)^3\cdot u'(x)$

$\frac{d}{dx}u(x)^5 = 5u(x)^4\cdot u'(x)$

product

Consequences

$$\frac{d}{dx}(u(x))^n = n(u(x))^{n-1}\frac{du(x)}{dx},\quad n\neq 0$$

$$\frac{d}{dx}\sin(u(x)) = \cos(u(x))\frac{du(x)}{dx}$$

$$\frac{d}{dx}\cos(u(x)) = -\sin(u(x))\frac{du(x)}{dx}$$

Others?

$\frac{d}{dx}\tan(u(x)) = \sec^2(u(x))\,u'(x)$

$\vdots$

Example: Give the derivative of $f(x) = \sin(3x) - 3\tan(x^2)$.

$f'(x) = \cos(3x)\cdot 3 - 3\sec^2(x^2)\cdot 2x$

$= 3\cos(3x) - 6x\sec^2(x^2)$

Example: Give the derivative of $g(x)=\sin^2(2x)-2\tan^3(3x)$.

$$g(x)=(\sin(2x))^2-2(\tan(3x))^3$$

$$g'(x)=2\sin(2x)\cdot\frac{d}{dx}\sin(2x)-2\cdot 3\tan^2(3x)\frac{d}{dx}\tan(3x)$$

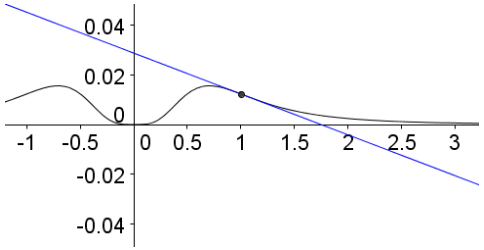
$$=2\sin(2x)\cdot\cos(2x)\cdot 2-6\tan^2(3x)\sec^2(3x)\cdot 3$$

$$=4\sin(2x)\cos(2x)-18\tan^2(3x)\sec^2(3x)$$

Example: Give the derivative of $f(x)=\left(\frac{x}{2x^2+1}\right)^4$

See the video.

What is the equation of the tangent line at $x=1$?



$f(x)=\left(\frac{x}{2x^2+1}\right)^4$ and its tangent line at $x=1$.

Popper P05

3. $f(x)=\sin(3x)-3\cos(2x)$. $f'\left(\frac{\pi}{6}\right)=$

(give your answer in decimal form)

4. $2+1=$

Example: Suppose $G(x) = f(v(x))$, $v(1) = 2$, $f'(1) = 3$, $f'(2) = -6$, and $v'(1) = 7$. Find $G'(1)$.

$G(x) = f(v(x))$

1. $G'(x) = f'(v(x))\,v'(x)$ Chain rule

2. $G'(1) = f'(v(1))\,v'(1)$

$= f'(2) \cdot 7$

$= (-6) \cdot 7 = -42$