Notes

- Check the Course Calendar for Homework, EMCF and Quiz information.
- Practice Test 2 is available. Your score counts as a quiz grade.
- Test 2 is Oct 4 - Oct 8. You should have already registered on CourseWare.

23 Video help for section 3.7	24 Blank Slides EMCF11 due at 9am Homework 4 Due in Lab/Workshop Quiz 3 Closes (2.6 and 3.1)	25	26 Blank Slides EMCF12 due at 9am Homework 5 Posted	27 Video help with section 3.8	28 EMCF13 due at 9am (a correction was made to problem 7 on 9/23) Last day to apply for fall graduation with a $25 fee.	29 Review Problems for Test 2
30	October 1 EMCF14 due at 9am Homework 5 due in lab/workshop Quiz 4 Closes (3.2-3.4)	2 **Online Live Review for Test 2 from 8:15-10:15pm. A link will appear here prior to the session.**	3	4 **Test 2 Starts**	5	6
7	**8** **Test 2 Ends** **Quiz 5 Closes** (3.5-3.6)	**9**	**10**	**11**	**12**	**13**

$s(0) = s_0$

Question: How does an object fall?

Assume the object falls at time $t = 0$ from a height s_0 and initial velocity v_0. ← velocity at $t=0$

Setting: low altitude - close to sea level

Ignore air friction.

* Assume: Gravitational force is constant.

$s(t)$ — height above ground level

Important Terms: Position, velocity, speed, and acceleration.

velocity: change in position wrt time, $v(t) = s'(t)$

speed: magnitude of velocity, $|v(t)|$

acceleration: change in velocity wrt time, $a(t) = v'(t)$

feet	meters
$a(t) = -32 \text{ ft/sec}^2$	$a(t) = -9.8 \text{ m/sec}^2$
$v(t) = -32t + C$	⋮
use $v(0) = v_0$: $v(t) = -32t + v_0$	$v(t) = -9.8t + v_0$
$s(t) = \underline{-16}\,t^2 + \underline{v_0}\,t + \underline{s_0}$	$s(t) = -4.9t^2 + v_0 t + s_0$

Hold until the object strikes the ground.

$V_0 = 0$ $\qquad$ $s_0 = 20$ ft

Example: An object is dropped from a height of 20 feet. If we neglect air friction, how long will it take for the object to hit the ground?

Solve $s(t) = 0$ for $t > 0$.

$s(t) = -16t^2 + 0t + 20 = -16t^2 + 20$

Solve $s(t) = 0$ for $t > 0$.

$-16t^2 + 20 = 0$ for $t > 0$

$t = \frac{\sqrt{5}}{2} \approx 1.118$ sec

Q: What is speed on impact?

A: speed $= |v(t)|$

$v(t) = s'(t) = -32t$

$\Rightarrow$ speed $= \left|v\left(\frac{\sqrt{5}}{2}\right)\right| = \left|-32\frac{\sqrt{5}}{2}\right| = 16\sqrt{5}\ \frac{\text{ft}}{\text{sec}}$

≈ 35.78 ft/sec

nearly 24.4 miles/hr.

Dropped ⟺ $v_0 = 0$

thrown upwards ⟺ $v_0 > 0$

or

thrown downwards ⟺ $v_0 < 0$

$s_0 = 20$

Example: An object is launched from a height of 20 feet. Give the initial velocity required to cause the object to strike the ground 5 seconds later.

↳ Find v_0 so that $s(5) = 0$.

$$s(t) = -16t^2 + v_0 t + 20$$

$$s(5) = 0 \iff -16 \cdot 25 + 5v_0 + 20 = 0$$

Solve for v_0.

$$v_0 = 76 \text{ ft/sec.}$$

Q: What is the speed on impact?

A: Find $|v(5)|$.

$$v(t) = s'(t) = -32t + 76$$

$$\Rightarrow |v(5)| = |-32 \cdot 5 + 76|$$

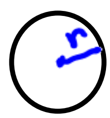
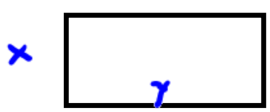
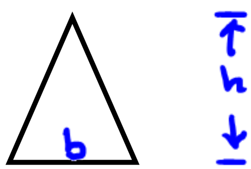
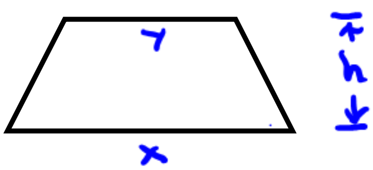
$$= |-84| = 84 \text{ ft/sec.}$$

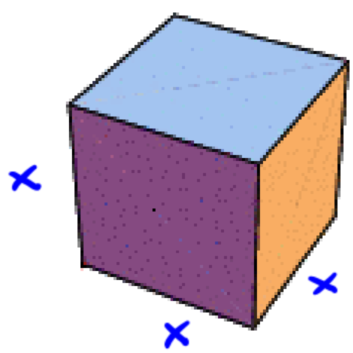
Rates of Change

You must know...

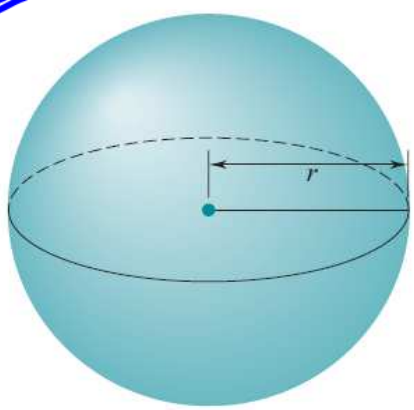
Areas, circumferences, volumes and surface areas of basic shapes.

Pythagorean Theorem

Shape	Area	Circumference
	πr^2	$2\pi r$
	xy	$2x + 2y$
Triangle 	$\frac{1}{2}hb$	Not Needed
Trapezoid 	$\frac{1}{2}(x+y)h$	Not Needed

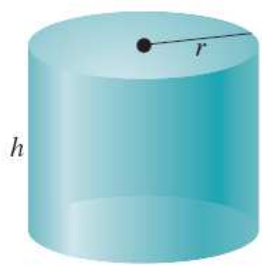
<table>
<tr><th>Shape</th><th>Volume</th><th>Surface Area</th></tr>
<tr><td>Cube


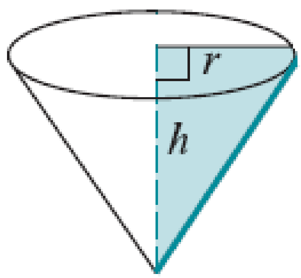
6 sides</td><td>x^3</td><td>$6x^2$</td></tr>
</table>

Shape	Volume	Surface Area
Sphere 	$\frac{4}{3}\pi r^3 = V$	$4\pi r^2 = S$

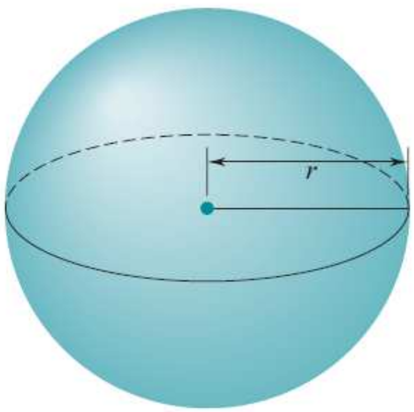
Note: $\frac{dV}{dr} = S$

Special for spheres.

Shape	Volume	Surface Area
right circular cylinder 	$\pi r^2 h$	$2\pi r^2 + 2\pi r h$

Shape	Volume	Surface Area
right circular cone 	$\frac{1}{3}\pi r^2 h$	Not Needed

Example: Give the rate of change of the surface area of a sphere with respect to its radius r.



$$S = 4\pi r^2$$

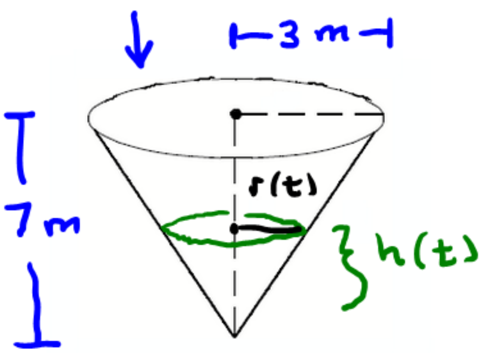
$$\frac{dS}{dr} = 8\pi r$$

Give the rate of change of the volume of a sphere with respect to its radius r.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

Example: A water tank in the shape of a right circular cone (with point down) is being filled with water. The height of the cone is 7 meters and the radius of the top of the tank is 3 meters. Suppose water is being added to the tank at the rate of 1/10 m^3/sec. How fast is the depth of the water in the tank increasing when the tank contains 50 m^3?

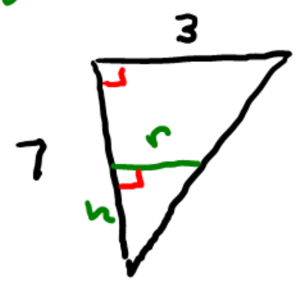


$V(t) \equiv$ volume at time t of water in the tank. (m^3, sec)

$V'(t) = \frac{1}{10}\ m^3/sec$

Find $h'(t)$ when volume is 50 m^3.

$V(t) = \frac{1}{3}\pi\, r(t)^2\, h(t)$



Similar triangles $\Rightarrow$

$\frac{h}{7} = \frac{r}{3} \Rightarrow r = \frac{3}{7}h$

$V(t) = \frac{1}{3}\pi\left(\frac{3}{7}h(t)\right)^2 h(t)$

$\Rightarrow$ (drop the "of t" part)

$V = \frac{1}{3}\pi \cdot \frac{9}{49}h^3 = \frac{3\pi}{49}h^3$

Diff wrt t.

$\frac{dV}{dt} = \frac{9\pi}{49}h^2\frac{dh}{dt}$

$\frac{dV}{dt}$ " $\frac{1}{10}$

Find $\frac{dh}{dt}$ when $V = 50$.

$$V = 50 \Rightarrow \frac{3\pi}{49} h^3 = 50$$

$$\Rightarrow h^3 = \frac{50 \cdot 49}{3\pi}$$

$$\Rightarrow h = \sqrt[3]{\frac{2450}{3\pi}} = \left(\frac{2450}{3\pi}\right)^{1/3}$$

$$\frac{1}{10} = \frac{9\pi}{49} \left(\frac{2450}{3\pi}\right)^{2/3} \frac{dh}{dt}$$

Solve for $\frac{dh}{dt}$.