Reminders

- Practice Test 2 is posted. Take the practice test early and often!
- You should have already registered for Test 2.
- There will be an online review for Test 2 next Tuesday night.

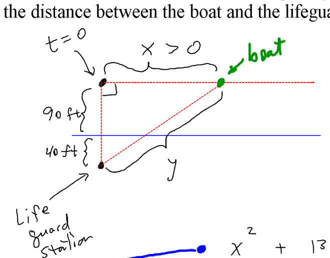
Related Rates

3.8

(Applied Chain Rule and Implicit Differentiation)

Setting: Two or more quantities are related by equation(s). All but one of the rates of change are known, and you are asked to find the other rate of change.

Example: A small boat is 90 feet offshore, and moving parallel to a straight beach. The boat is moving at a constant speed of 10 feet per second. At time $t = 0$, the boat is directly opposite a lifeguard station which is 40 feet from the water. How fast is the boat moving away from the lifeguard station when the distance between the boat and the lifeguard station is 150 feet?



$\frac{dx}{dt} = 10 \text{ ft/sec.}$

Find $\frac{dy}{dt}$ when $y = 150$ ft.

$x^2 + 130^2 = y^2$

Diff wrt t. (Note: x and y depend upon t)

$\frac{d}{dt}(x^2 + 130^2) = \frac{d}{dt} y^2 \Rightarrow \cancel{2}x\frac{dx}{dt} + 0 = \cancel{2}y\frac{dy}{dt}$

$x \frac{dx}{dt} = y \frac{dy}{dt}$

??   10   ← Find when y = 150

$x^2 + 16900 = 150^2$

$x^2 + 16900 = 22500$

$x^2 = 5600$

$x = \sqrt{5600}$

$= 20\sqrt{14}$

$20\sqrt{14} \cdot 10 = 150 \frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{4}{3}\sqrt{14}$ ft/sec

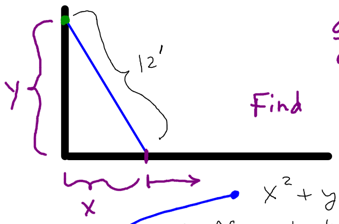
$\approx 4.98\ldots$ ft/sec.

Popper P08

1. Suppose x and y are differentiable functions of t and $x^2 + y^2 = 25$. If $dx/dt = 10$ ft/sec, then dy/dt = _______ ft/sec when $x = 3$.

2. Suppose x and y are differentiable functions of t and $x^2 + y^2 = 25$. If $dy/dt = 4$ ft/sec, then dx/dt = _______ ft/sec when $x = 4$.

Example: A 12 foot ladder is leaning against a wall. If the base of the ladder is moving away from the wall at the rate of 1 foot per second, at what rate will the top of the ladder be moving when the base of the ladder is 5 feet from the wall?



$\frac{dx}{dt} = 1$ ft/sec.

Find $\left|\frac{dy}{dt}\right|$ when $x = 5'$

$x^2 + y^2 = 144$

Diff wrt t. (Note: x and y are functions of t)

$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}144$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

1 ?

Find when $x = 5$

$25 + y^2 = 144$

$y^2 = 119$

$y = \sqrt{119}$

Plug in. $\frac{dy}{dt} = \frac{-5}{\sqrt{119}}$

$\Rightarrow \left|\frac{dy}{dt}\right| = \frac{5}{\sqrt{119}}$ ft/sec.

$\approx .45\ldots$ ft/sec.

Popper P08

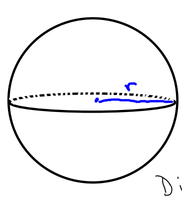
3. A 12 foot ladder is leaning against a wall. If the base of the ladder is moving away from the wall at the rate of 2 ft/sec, at what rate will the top of the ladder be moving when the base of the ladder is 5 feet from the wall?

4. A 12 foot ladder is leaning against a wall. If the top of the ladder is moving down the wall at the rate of 2 ft/sec, at what rate will the base of the ladder be moving away from the wall when the base of the ladder is 5 feet from the wall?

Example: A large spherical balloon is inflated so that its volume is increasing at the rate of 3 cubic feet per minute.

- 1. How fast is the radius of the balloon increasing at the instant when the diameter of the balloon is 1 foot?
- How fast is the surface area of the balloon increasing at the instant when the diameter is 1 foot?

V



$\frac{dV}{dt} = 3$ ft^3/min

Find $\frac{dr}{dt}$ when $r = \frac{1}{2}'$

$V = \frac{4}{3}\pi r^3$

Diff wrt t. Note: V and r are functions of t.

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$3 =$

Find when $r = \frac{1}{2}$

$\Rightarrow \frac{dr}{dt} = \frac{3}{\pi}$ ft./min.

See the video for the rest.

Example: Sand is falling onto a conical pile so that the radius of the base of the pile is always equal to one half if its altitude. If the sand is falling at a rate of 6 cubic feet per minute, how fast is the altitude of the pile increasing when the pile is 9 feet deep?

See the video for the rest.