

### Test 2 Review

- Evaluating limits of basic functions, from formulas and graphs.
- Evaluating  $\sin(u)/u$  type limits.
- Determining continuity of functions from formulas and graphs.
- Intermediate value theorem.
- Using the definition of derivative to find a derivative.
- Using a graph to determine where a derivative does not exist.
- Using the power, product and quotient rules to find derivatives.
- Derivatives of trigonometric functions.
- Chain rule.
- Tangent and normal lines.
- Implicit differentiation.
- Rates of change and related rates.

Test 2 will not contain any problems related to  $\epsilon\delta$  proofs, the extreme value theorem or the pinching theorem.

Someone Has a Birthday Today!

Sep 28-10:14 AM

### Good Sources of Practice Problems

- The textbook
- Homework
- Poppers
- EMCFs
- Online Quizzes
- Online Practice Test 2
- Lab/workshop quizzes
- Class examples
- This problem set
- The other posted review set
- Next Tuesday's review

**Examples:**  $\lim_{x \rightarrow 3} (3x^2 - 5x + 2) =$

$$\lim_{x \rightarrow 1} \frac{\sin(3x)}{2x} =$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x - 2}{x^2 - 1} =$$

Video

**Examples:**  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x + 2} =$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} =$$

$$\lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(3x)} =$$

$$\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos(x)} =$$

Video

## Popper P09

1.  $\lim_{x \rightarrow 0} \frac{\sin(2x)}{5x} =$

2.  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 5x + 6} =$

**Example:** Give values of  $A$  and  $B$  so that  $g(x) = \begin{cases} Ax + B & x < 2 \\ 2 & x = 2 \\ Ax^2 - Bx & x > 2 \end{cases}$

is continuous.

The only issue is  $x = 2$ .

Need:

$$g(2) = \lim_{x \rightarrow 2} g(x) \begin{cases} \lim_{x \rightarrow 2^-} (Ax + B) = 2A + B \\ \lim_{x \rightarrow 2^+} (Ax^2 - Bx) = 4A - 2B \end{cases}$$

$$\begin{aligned} 2(2A + B) &= 2 \\ + (4A - 2B) &= 2 \\ \hline 8A &= 6 \Rightarrow A = 3/4 \\ 4(3/4) - 2B &= 2 \Rightarrow B = 1/2 \end{aligned}$$

**Example:** Describe the discontinuities of the function  $f(x) = \frac{x+2}{x^2+4x+4}$ .

$\left. \begin{array}{l} \vdots \\ \vdots \end{array} \right\} \text{rational function}$

$f$  is discontinuous where

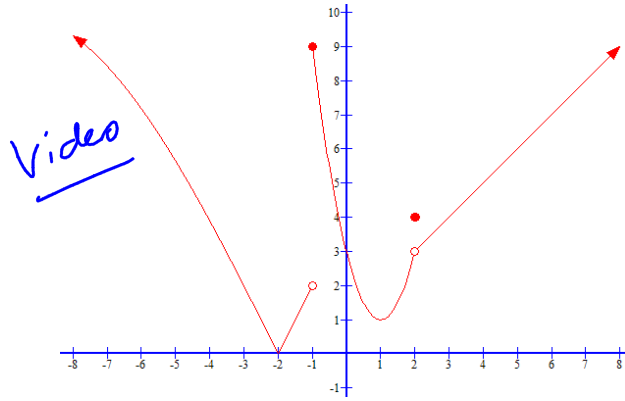
$$\begin{aligned} x^2 + 4x + 4 &= 0 \\ (x+2)^2 &= 0 \\ x &= -2 \end{aligned}$$

Describe:  $f(x) = \frac{x+2}{(x+2)^2} = \frac{1}{x+2}$

vert asympt at  $x = -2$

$\therefore f$  has an infinite discat at  $x = -2$ .

**Example:** The function  $f$  is shown below. Describe any discontinuities.



**Example:** Use the intermediate value theorem to prove there is a solution to  $3x^3 - 2x^2 - 2x = 1$  on the interval  $[-1, 3]$ .

Video

**Examples:**  $\frac{d}{dx} \left( \frac{2x-1}{3x^2+4x} \right) =$

$\frac{d}{dx} (\sin(3x)\cos(2x)) =$

$f(x) = 2(x^2+1)^3$ . Give  $f'''(x)$ .

Video

**Example:**  $G(x) = (f \circ g)(x)$ . Suppose  $f(2) = 3$ ,  $f'(2) = 3$ ,  $f(1) = -2$ ,  $f'(1) = -4$ ,  $g(1) = 2$ , and  $g'(1) = -3$ . Give  $G'(1)$ .

$$G(x) = f(g(x))$$

1.  $G'(x) = f'(g(x))g'(x)$
2.  $G'(1) = f'(g(1))g'(1)$   
 $= f'(2)(-3)$   
 $= 3(-3) = -9$

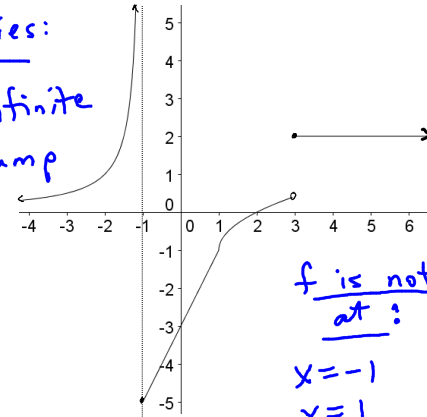
**Example:**  $G(x) = \sin(\pi f(x)) + (2g(x)-1)^2$ . Suppose  $f(2)=3, f'(2)=3,$   
 $f(1)=-2, f'(1)=-4, g(1)=2,$  and  $g'(1)=-3$ . Give  $G'(1)$ .

Video

**Example:** The function  $f$  is shown below. Describe any discontinuities. Also, give the values where the function is not differentiable.

Discontinuities:

$x = -1$  infinite  
 $x = 3$  jump



f is not diff. at:

$x = -1$   
 $x = 1$  corner  
 $x = 3$

$g(x) = \frac{a}{bx+c}, g(x) = \sqrt{ax+b},$   
 $g(x) = ax^2+bx+c$

**Example:** Use the definition of derivative to find the derivative

of  $f(x) = \frac{3}{2x-5}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

"0/0" ind. form

$$= \lim_{h \rightarrow 0} \frac{\frac{3}{2(x+h)-5} - \frac{3}{2x-5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{3(2x-5) - 3(2(x+h)-5)}{(2(x+h)-5)(2x-5)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6x - 15 - 3(2x + 2h - 5)}{h(2(x+h)-5)(2x-5)}$$

$$= \lim_{h \rightarrow 0} \frac{-6h}{h(2(x+h)-5)(2x-5)}$$

$$= \frac{-6}{(2x-5)^2}$$

**Example:** Give a formula for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ , given that

$x^3 - 3xy + 2y^3 = 2$ . Then give the equation of the normal line to the graph at the point (0,1).

Treat  $y$  like a diff function of  $x$ .  
 Diff w.r.t  $x$ .

$$\frac{d}{dx}(x^3 - 3xy + 2y^3) = \frac{d}{dx} 2$$

$$3x^2 - [3x \frac{dy}{dx} + y \cdot 3] + 6y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(-3x + 6y^2) = -3x^2 + 3y$$

$$\frac{dy}{dx} = \frac{-x^2 + y}{-x + 2y^2}$$

N.L. at (0,1): slope =  $\frac{-1}{-x + 2y^2} \Big|_{(0,1)} = -2$   
 point = (0,1)

Equation:  
 $y - 1 = -2x$   
 or  $y = -2x + 1$ .

**P09**

3. Give the slope of the tangent line in the previous example at the point  $(0,1)$ .
4. Give the  $y$ -intercept of the tangent line in the previous example at the point  $(0,1)$ .

**Example:** An object is thrown upwards with an initial velocity of 20 ft/sec, and it strikes the ground 5 seconds later. Give the initial height of the object, and the speed of the object on impact. (Neglect air friction.)

Video

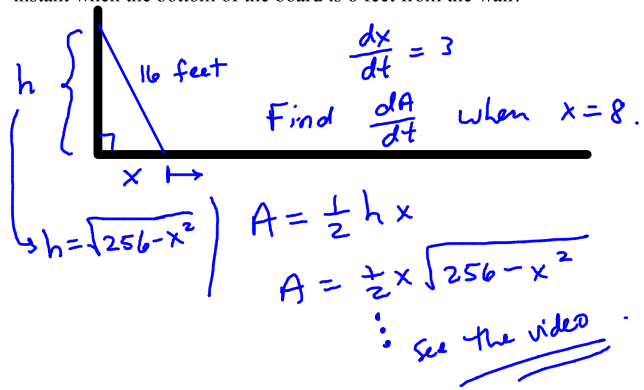
**Example:** A particle moves along the real line so that its position at time  $t$  seconds is given by  $x(t) = t - 3t^2 + \cos(2\pi t)$  feet. Give the velocity and speed of the particle when  $t = 1/3$ .

Video

**Example:** A 5 foot tall girl is walking towards a 25 foot lamp post at the rate of 2 feet per second. How fast is the tip of her shadow moving when she is 10 feet from the lamp post?

Video

**Example:** A 16 foot board is leaning against a vertical wall. If the bottom of the board slides away from the wall at the rate of 3 feet per second, how fast is the area of the triangle formed by the board, the floor and the wall changing at the instant when the bottom of the board is 8 feet from the wall?



$$h = \sqrt{256 - x^2}$$
$$A = \frac{1}{2} h x$$
$$A = \frac{1}{2} x \sqrt{256 - x^2}$$

∴ see the video