Info

- **EMCFs are due every MWF morning.**
- **There is a quiz in lab Friday.**
- **There is no homework due on Monday.**
- **There will be an EMCF due on Monday.**
- **There is an online quiz due Monday.**
- **Practice Test 2 is posted.**
- **The slides and video are posted from last night's review.**
- **You should be registered for Test 2.**

The **ALD Honor Society** will have a general meeting at 5:30 ← Today
in CTC lab room 239.
Go see what it takes to become a member.

more

Differentials and Newton's Method

Section 3.9

(tangent line approximation)

recall

Newton's Method - Formula

Approximating sol'ns to $f(x)=0$.

Let f be a twice differentiable function and suppose a is a real number at which $f(a) = 0$.

Sppse there's a true sol'n.

If $f'(a) \neq 0$ and x_0 is sufficiently close to a, then the iteration

Guess

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\vdots$$

will converge (rapidly) to the root a.

Example: Do one iteration of Newton's method from a guess of $\boxed{x_0 = 2}$ to approximate a solution to $x^4 + 2x - 3 = 0$. Then compute further Newton iterates using a calculator or other computing device.

Note: $x=1$ is a sol'n.

$f(x) = x^4 + 2x - 3$

$f'(x) = 4x^3 + 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{f(2)}{f'(2)} = 2 - \frac{17}{34} = 1.5$

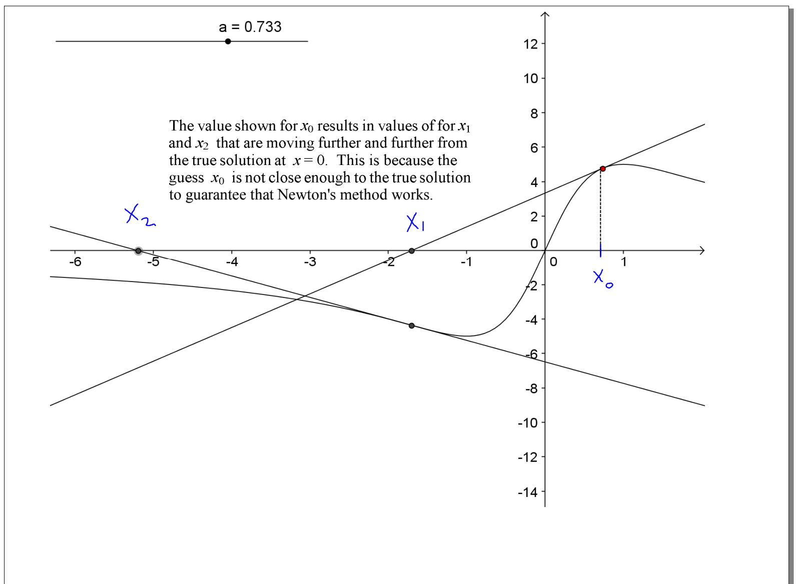
$f(2) = 17$

$f'(2) = 34$

	xi	f(xi)
$x_0 =$	2.00000000	17.00000000
$x_1 =$	1.50000000	5.06250000
$x_2 =$	1.17338710	1.24245509
$x_3 =$	1.02656388	0.16369262
⋮	1.00069307	0.00416132
	1.00000048	0.00000288
	1.00000000	0.00000000
	1.00000000	0.00000000
	1.00000000	0.00000000
	1.00000000	0.00000000
	1.00000000	0.00000000

Example: Newton's method can go horribly wrong IF the initial guess is not sufficiently close to the actual solution. We can see this by exploring the equation $\boxed{\frac{10x}{x^2+1} = 0}$

Everyone can see $x=0$ solves.



P11

1. Use one iteration of Newton's method from a guess of $x_0 = \frac{3}{2}$ to approximate a solution to $x^2 - 3 = 0$.

recall

The differential of f at a with increment h is given by $df = f'(a)h$

x value

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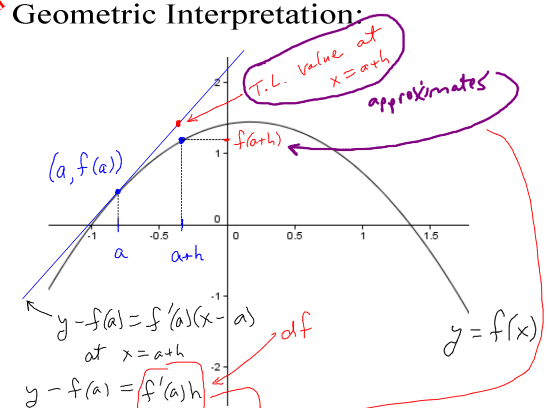
recall

Geometric Interpretation:



$y - f(a) = f'(a)(x-a)$

at $x = a+h$

$y - f(a) = \boxed{f'(a)h}$

$df \approx f(a+h) - f(a)$

$f(a+h) \approx f(a) + \underbrace{f'(a)h}_{df}$

Differentials Can Be Used To Approximate Function Values

The differential of f at a with increment h is given by $df = f'(a)h$

Using the approximation $df \approx f(a+h) - f(a)$, the equation above becomes

$$f(a+h) \approx f(a) + f'(a)h$$

(this is a tangent line approximation)

typically, $f(x)$ is known at a.

Quick and dirty approx.

Example: Use differentials to approximate $\sqrt{25.1}$.

25.1 is "close" to 25, and we know $\sqrt{25} = 5$.

$f(x) = \sqrt{x}$. $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$\sqrt{25.1} = f(\underset{a+h}{25.1}) \approx f(\underset{a}{25}) + f'(\underset{a}{25}) \cdot (\underset{h}{.1})$

$\approx 5 + \frac{1}{10} \cdot \frac{1}{10} = 5.01$

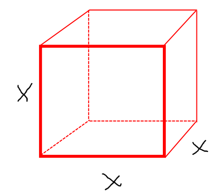
$\sqrt{24.9} = f(\underset{a+h}{24.9}) \approx f(\underset{a}{25}) + f'(\underset{a}{25})(\underset{h}{-.1})$

$\approx 5 + \frac{1}{10}\left(-\frac{1}{10}\right) = 4.99$

P11

2. Use differentials to approximate $\sqrt{36.1}$.

Example: A box is to be constructed in the form of a cube to hold 1000 cubic feet. Use a differential to estimate how accurately the edge must be made so that the volume will be correct to within 3 cubic feet.



$V = x^3$

$$\boxed{\begin{array}{l} V = 1000 \\ x = 10 \end{array}}$$

$dV = 3x^2 \cdot h$

max error is 3 ft^3

10

error in side length

Quick estimate of the allowable error

$3 = 300h$

$h = 0.01$ ft.