Info
- There is no Homework due next Monday.
- There are EMCFs due every MWF.
- There is an Online Quiz due Monday.
- Take care of Practice Test 2!!
- Schedule and take Test 2!!

The Mean Value Theorem
Section 4.1

Question: How many values are there between -1 and 3 where the tangent line is parallel to the secant line connecting (-1, f(-1)) and (3, f(3))?

Question: How many values are there between 0 and 6 where the tangent line is parallel to the secant line connecting (0, f(0)) and (6, f(6))?
**General Question:** Are there values between \( a \) and \( b \) where the tangent line is parallel to the secant line connecting \((a, f(a))\) and \((b, f(b))\)?

Yes. We see 2 values.

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

**Mean Value Theorem:** If \( f \) is continuous on \([a,b]\) and differentiable on \((a,b)\), then there is at least one value \( c \) between \( a \) and \( b \) so that

\[ f(b) - f(a) = f'(c) \cdot (b - a) \]

**Rolle’s Theorem:** Some conditions on \( f \). If \( f(a) = f(b) = 0 \) then there is at least one value \( c \) between \( a \) and \( b \) where

\[ f'(c) = 0. \]

**Example:** Verify the mean value theorem for \( f(x) = 3x - x^2 \) on the interval \([-1,3]\).

Find a value \( c \) between -1 and 3 so that

\[ f'(c) = \frac{f(3) - f(-1)}{3 - (-1)} \]

\[ 3 - 2c = \frac{3 - 4}{4} \]

\[ 3 - 2c = 1 \]

\[ 2c = 2 \]

\[ c = 1 \]

Note: \(-1 < 1 < 3\).

**Example:** How many values of \( x \) satisfy the conclusion of the Mean Value Theorem for \( f(x) = 3x + \sin(x) \) on the interval \([-1,8]\).

1. Algebraically, find all values \( c \) between -1 and 8 so that

\[ f'(c) = \frac{f(8) - f(-1)}{8 - (-1)} \]

\[ f(x) = 3 + \sin(x) \]

\[ f(8) = 3 + \sin(8) \approx 3.203 \]

\[ 3 + \sin(c) = 3.203 \]

\[ 0.203 > 0 \]

\[ -1 < c < 8 \]

3 values

\[ c = 0, 2.5 \]

Note: \(-1 < c < 8\).