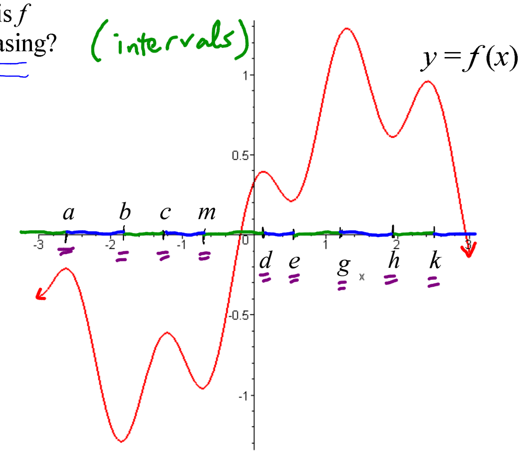
Info...

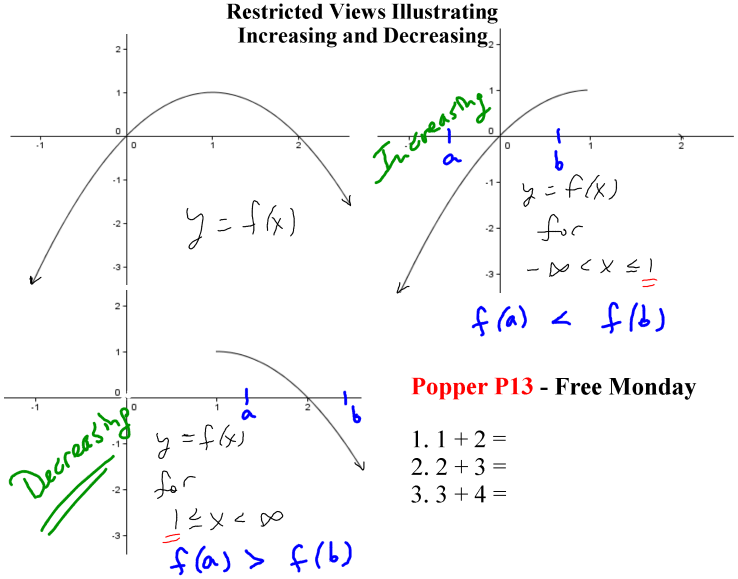
- We will cover portions of 4.2 and 4.3 today.
- Homework and EMCFs are posted.
- Online Quiz 5 is due tonight at 11:59pm.
- Practice Test 2 is due tonight at 11:59pm.
- Please complete Online Quizzes 6 and 7 asap.
- Today is the last day to take Test 2.

Increasing and Decreasing Functions

Intuitively, where is f increasing? Decreasing? (intervals)



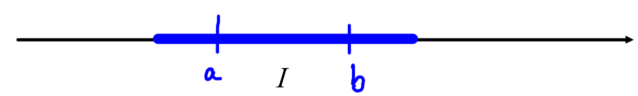
Restricted Views Illustrating Increasing and Decreasing



Popper P13 - Free Monday

1. 1 + 2 =
2. 2 + 3 =
3. 3 + 4 =

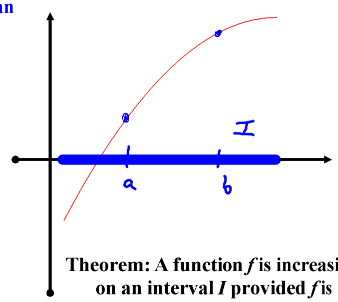
Algebraic Definitions of Increasing and Decreasing on an Interval



f is increasing on I if and only if $f(a) < f(b)$ whenever $a, b \in I$ and $a < b$.

f is decreasing on I if and only if $f(a) > f(b)$ whenever $a, b \in I$ and $a < b$.

Definition: *f* is increasing over an interval *I* if and only if
$f(a) < f(b)$
for all *a*, *b* in *I* with $a < b$.

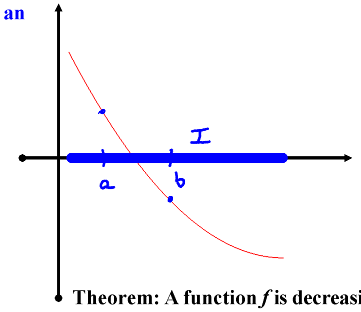


Theorem: A function *f* is increasing on an interval *I* provided *f* is continuous and
$f'(x) > 0$
at all but finitely many values in *I*.

What property does the derivative have on this interval?

mostly positive

Definition: *f* is decreasing over an interval *I* if and only if
$f(a) > f(b)$
for all *a*, *b* in *I* with $a < b$.



Theorem: A function *f* is decreasing on an interval *I* provided *f* is continuous and
$f'(x) < 0$
at all but finitely many values in *I*.

What property does the derivative have on this interval?

mostly negative

Example: Determine the intervals of increase and decrease for the function $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x - 3$

Important Idea: Slope chart.

Popper P13 - Free Monday

4. $1 - 2 =$
5. $2 - 3 =$
6. $3 - 4 =$

polynomial (continuous)

$f'(x) = x^2 + 5x + 6$

Set $f'(x) = 0$

$x^2 + 5x + 6 = 0 \iff (x+2)(x+3) = 0$

$x = -2$ or $x = -3$.

Slope Chart

$f'(x)$ + + + + + + 0 − − − − − 0 + + + + +

x −4 −3 $-\frac{5}{2}$ −2 −1

$f'(-4) = (-)(-) = +$ $f'(-\frac{5}{2}) = (-)(+) = -$ $f'(-1) = (+)(+) = +$

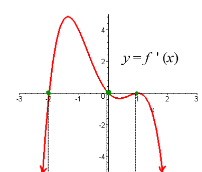
f is increasing on $(-\infty, -3]$ and $[-2, \infty)$

f is decreasing on $[-3, -2]$.

round brackets will be accepted EVEN THOUGH THEY ARE WRONG.

Example: The graph of $y = f'(x)$ is shown below. Give the interval(s) on which f is increasing, and the interval(s) on which f is decreasing.

Not the graph of f.



slope chart

$f'(x)$ − − − 0 + + + 0 − − 0 − − − − − − − −

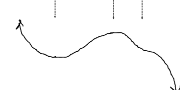
x −2 0 1

f is increasing on $[-2, 0]$.

f is decreasing on $(-\infty, -2]$, $[0, \infty)$

Question: Can you determine the shape of the graph of f?

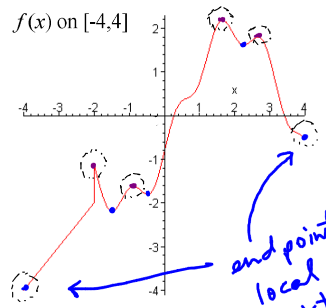
rough shape



local maximums and minimums

Local vs Absolute Extrema

local highs and lows

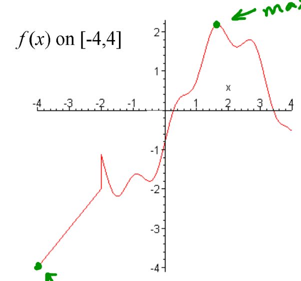


end point local extrema.

Question: Can you identify the local extreme values of this function?

yes

Question: In general, what does it mean to say that a function f has a local extreme value at $x = c$?



Question: Can you identify the absolute extreme values of this function?

maximum or minimum (Not local)

Question: In general, what does it mean to say that a function f has an absolute extreme value at $x = c$?

Question: Does every function have a largest value?

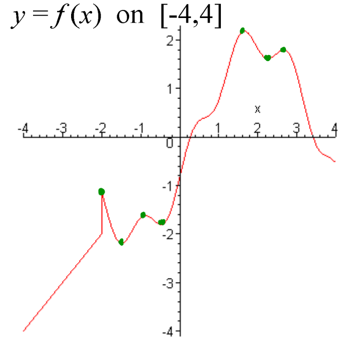
No. Ex. $f(x) = x$ for $-\infty < x < \infty$

Ex. $g(x) = \begin{cases} 1/x, & -1 \le x < 0 \text{ or } 0 < x \le 1 \\ 7, & x = 0 \end{cases}$

Theorem: A continuous function f on a closed bounded interval $[a,b]$ has both an absolute maximum value and an absolute minimum value on the interval $[a,b]$.

This is the Extreme Value Theorem!!

Remark: If no interval is specified, then we have to assume that all values of x are valid, so long as they can be put in the function.



Question: What will be true about f' at a value of x where f has a local extreme value?

$f'(x) = 0$

or

$f'(x)$ dne

Note: These values of x are so important that we give them a special name... Critical Numbers.

Critical Numbers

The value $x = a$ is a critical number for f if and only if a is in the domain of f and either $f'(a) = 0$ or $f'(a)$ does not exist.

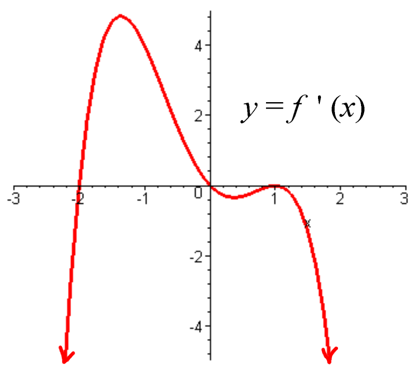
$x = a$

$(a, f(a))$

We can classify critical numbers and critical points as either local maximums or local minimums by using the slope chart.

This is called the first derivative test.

Example: The graph of $y = f'(x)$ is shown below. Classify the critical numbers of f.



Example: Find the critical numbers for the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 2$.

Then classify each of the critical numbers as places where the function has a local minimum, local maximum or neither.

$f'(x) = x^2 + x - 6$ ← polynomial defined for all x.

Set $f'(x) = 0$

$x^2 + x - 6 = 0 \iff (x+3)(x-2) = 0$

$\underbrace{x = -3,\ x = 2}_{\text{critical numbers.}}$

slope chart

$f'(x)$: + + + + 0 − − − − 0 + + + +

x: −4, −3, 0, 2, 3

$f'(-4) = +$ $\quad f'(0) = -$ $\quad f'(3) = +$

Local shape for f

local max at $x = -3$

local min at $x = 2$