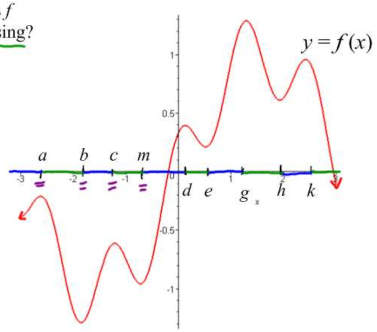
Info...

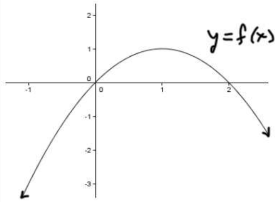
- We will cover portions of 4.2 and 4.3 today.
- Homework and EMCFs are posted.
- Online Quiz 5 is due tonight at 11:59pm.
- Practice Test 2 is due tonight at 11:59pm.
- Please complete Online Quizzes 6 and 7 asap.
- Today is the last day to take Test 2.

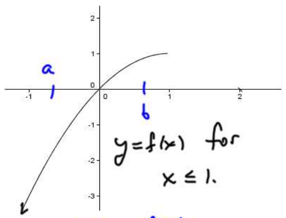
Increasing and Decreasing Functions

Intuitively, where is f increasing? Decreasing?



Restricted Views Illustrating Increasing and Decreasing

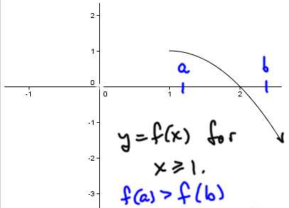




$y=f(x)$ for $x \le 1$.

$f(a) < f(b)$

f is increasing on $(-\infty, 1]$

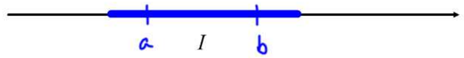


$y=f(x)$ for $x \ge 1$.

$f(a) > f(b)$

f is decreasing on $[1, \infty)$.

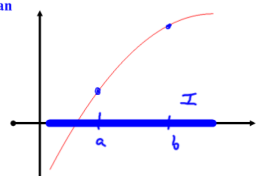
Algebraic Definitions of Increasing and Decreasing on an Interval



f is increasing on I if and only if when $a, b \in I$ and $a < b$, we have $f(a) < f(b)$

f is decreasing on I if and only if when $a, b \in I$ and $a < b$, we have $f(a) > f(b)$

Definition: f is **increasing over an interval I** if and only if
$f(a) < f(b)$
for all a, b in I with $a < b$.

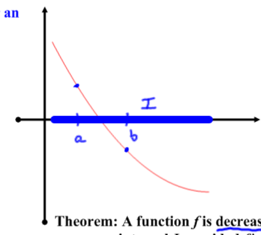


What property does the derivative have on this interval?

mostly positive

Theorem: A function f is increasing on an interval I provided f is continuous and
$f\,'(x) > 0$
at all but finitely many values in I.

Definition: f is **decreasing over an interval I** if and only if
$f(a) > f(b)$
for all a, b in I with $a < b$.



What property does the derivative have on this interval?

mostly negative

Theorem: A function f is decreasing on an interval I provided f is continuous and
$f\,'(x) < 0$
at all but finitely many values in I.

Example: Determine the intervals of increase and decrease for the function $f(x)=\frac{1}{3}x^3+\frac{5}{2}x^2+6x-3$

Important Idea: Slope chart.

$f'(x)$ is continuous

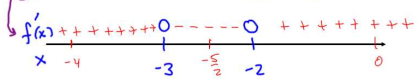
$f'(x) = x^2+5x+6$

Set $f'(x)=0 \Leftrightarrow x^2+5x+6=0$

$(x+2)(x+3)=0$

$x=-2,\ x=-3$

Slope chart



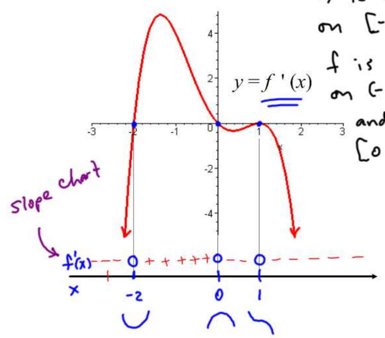
$f'(-4)=(-)(-) = +$ $\quad$ $f'(-\frac{5}{2})=(-)(+) = -$ $\quad$ $f'(0)=+$

f is increasing on $(-\infty,-3]$ and $[-2,\infty)$

f is decreasing on $[-3,-2]$

rough shape of f:

Example: The graph of $y=f\,'(x)$ is shown below. Give the interval(s) on which f is increasing, and the interval(s) on which f is decreasing.



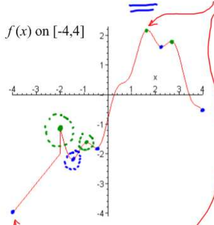
f is increasing on $[-2,0]$

f is decreasing on $(-\infty,-2]$ and $[0,\infty)$.

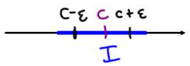
Question: Can you determine the shape of the graph of f?

rough shape

Local vs Absolute Extrema

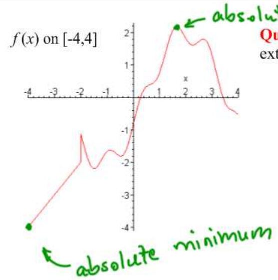


Question: Can you identify the local extreme values of this function?



Question: In general, what does it mean to say that a function f has a local extreme value at $x = c$?

Spse we are considering f on an interval I containing c. f has a local maximum (minimum) at $x = c$ iff there is a value $\varepsilon > 0$ so that if $x \in I$ and $|x - c| < \varepsilon$ then $f(c) \geq f(x)$ ($\leq$).



Question: Can you identify the absolute extreme values of this function?

Question: In general, what does it mean to say that a function f has an absolute extreme value at $x = c$?

Consider f on an interval I. f has an absolute maximum (minimum) at $c \in I$ if and only if $f(c) \geq f(x)$ ($\leq$) for all $x \in I$.

Question: Does every function have a largest value?

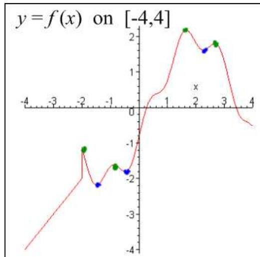
No: $f(x) = x$ for $-\infty < x < \infty$.

$$g(x) = \begin{cases} 1/x, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Theorem: A continuous function f on a closed bounded interval $[a,b]$ has both an absolute maximum value and an absolute minimum value on the interval $[a,b]$.

This is the Extreme Value Theorem!!

Remark: If no interval is specified, then we have to assume that all values of x are valid, so long as they can be put in the function.



Question: What will be true about f' at a value of x where f has a local extreme value?

Either $f'(x) = 0$ or $f'(x)$ does not exist.

Note: These values of x are so important that we give them a special name... Critical Numbers.

Critical Numbers

The value $x = a$ is a critical number for f if and only if a is in the domain of f and either $f'(a) = 0$ or $f'(a)$ does not exist.

$(a, f(a))$ where a is a critical number

We can classify critical points as either local maximums or local minimums by using the slope chart.

This is called the first derivative test.

Example: Find the critical numbers for the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 2$.
Then classify each of the critical numbers as places where the function has a local minimum, local maximum or neither.

$f'(x) = x^2 + x - 6$ ← polynomial ← continuous

$f'(x)$ exists for all x.

set $f'(x) = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0 \Leftrightarrow x = -3$ or $x = 2$

$x = -3$ and $x = 2$ are critical numbers.

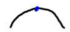
slope chart

$f'(x)$ + + + + + + 0 − − − − − − 0 + + + +

x -4 -3 0 2 3

$f'(-4) = (-)(-) = +$ $f'(0) = -$ $f'(3) = (+)(+) = +$

Basic shape for f



local max at $x = -3$

local min at $x = 2$

We can classify critical points as either local maximums or local minimums by using the slope chart.

This is called the first derivative test.