Information

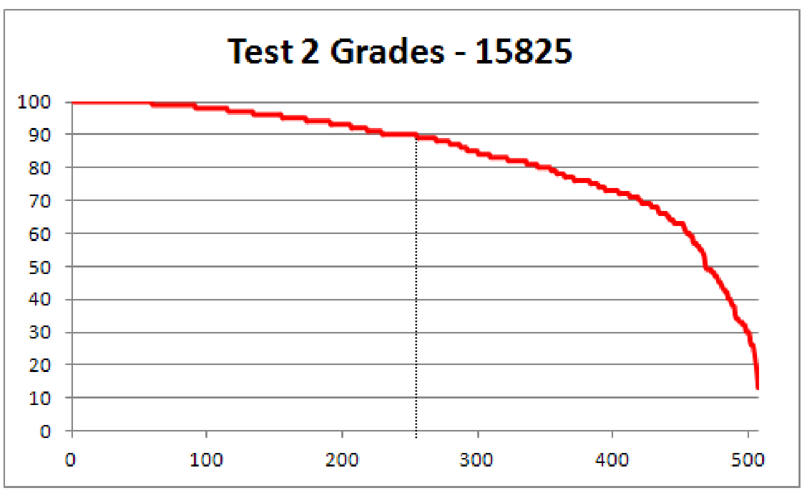
- EMCFs are due MWF.
- Homework is posted for Monday, and an online quiz is due Monday night.
- The written portion of Test 2 is graded, and the grades for the multiple choice and written portions appear in a separate entries in the gradebook. You will need to add the scores to get your total score out of 100.

Dealing with Distractions...

One moment please.

Test 2 Results

Section	Median	Mean	Number
15819	85	81.16	528
15825	90	83.32	507
15836	87	81.46	334
15841	77	73.4	137



A	254
B	100
C	67
D	36
F	50

Recall: Critical Numbers

Critical values

The value $x = c$ is a critical number for f if and only if c is in the domain of f and either $f'(c) = 0$ or $f'(c)$ does not exist.

$(c, f(c))$

How can we classify critical points as either local maximums or local minimums?

The First Derivative Test

slope chart.

Example: Use the first derivative test to classify the local extrema of the function

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 4$ ← polynomial

Domain of f is $(-\infty, \infty)$.

$f'(x) = 4x^3 - 24x^2 + 44x - 24$

↖ polynomial

$\therefore f'(x)$ exists for all x.

Set $f'(x) = 0$. $4x^3 - 24x^2 + 44x - 24 = 0$

$x^3 - 6x^2 + 11x - 6 = 0$

$x = 1$ solves this. ← c.n. for f.

$$
\begin{array}{r}
x^2 - 5x + 6 \\
x-1 \,\big)\, x^3 - 6x^2 + 11x - 6 \\
-\,(x^3 - x^2) \\
\hline
-5x^2 + 11x - 6 \\
-\,(-5x^2 + 5x) \\
\hline
6x - 6 \\
-\,(6x - 6) \\
\hline
0
\end{array}
$$

$(x-1)(x^2 - 5x + 6) = 0$

$(x-1)(x-2)(x-3) = 0$

$x = 1,\ x = 2,\ x = 3.$

$f'(x) = 4(x-1)(x-2)(x-3)$

slope chart:

$f'(x)$: − − − − 0 + + + + 0 − − − 0 + + + + +

x: 0 1 1.5 2 2.5 3 4

$f'(0) = -$ $f'(1.5) = +$ $f'(2.5) = -$ $f'(4) = +$

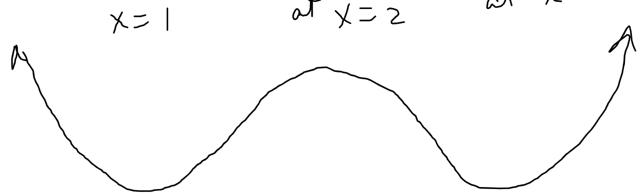
rough shape of f (local)

local min at $x = 1$

local max at $x = 2$

local min at $x = 3$

Global (estimate)



Popper P14

1. Give the largest critical value of $f(x) = 2x^3 - 3x^2 - 12x + 1$.

2. Give the smallest critical value of $f(x) = 2x^3 - 3x^2 - 12x + 1$.

3. Give the critical value of $f(x) = 2x^3 - 3x^2 - 12x + 1$ where a local maximum occurs.

4. Give the critical value of $f(x) = 2x^3 - 3x^2 - 12x + 1$ where a local minimum occurs.

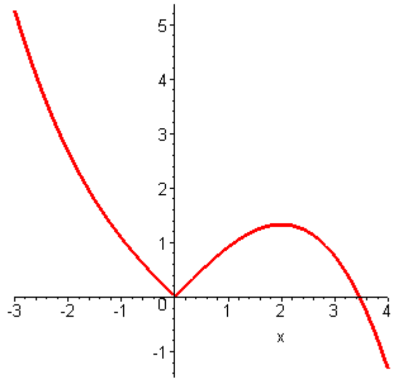
Example: Find and classify the critical numbers of

$$f(x) = |x| - \frac{x^3}{12}.$$

← Domain is $(-\infty, \infty)$.

corner at $x = 0$.

smooth at $x = 0$.



corner − smooth = corner.

i.e. $f'(0)$ dne. ✱

$x = 0$ is a c.n.

$$f(x) = \begin{cases} -x - \dfrac{x^3}{12}, & x < 0 \\ x - \dfrac{x^3}{12}, & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -1 - \frac{1}{4}x^2, & x < 0 \\ 1 - \frac{1}{4}x^2, & x > 0 \end{cases}.$$

Set $f'(x) = 0$.

⇕

Search $x < 0$ and $x > 0$.

$x < 0$: $\quad -1 - \frac{1}{4}x^2 = 0$

$1 + \frac{1}{4}x^2 = 0$. Impossible.

$x > 0$: $\quad 1 - \frac{1}{4}x^2 = 0$

$\cancel{x = -2}$ or $x = 2$ ✓

$x = 2$ is a c.n.

f has c.n. at $x = 0$ and $x = 2$.

Slope chart:

$f'(x)$	− − − −	corner dne	+ + + +	0	− − −
x	−1	0	1	2	3

$$f'(x) = \begin{cases} -1 - \frac{1}{4}x^2, & x < 0 \\ 1 - \frac{1}{4}x^2, & x > 0 \end{cases}$$

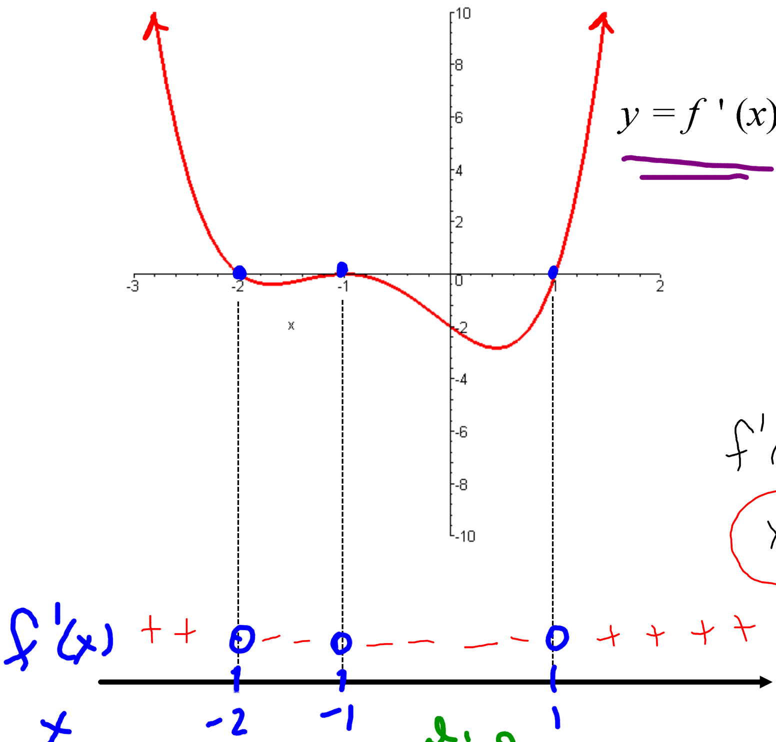
$f'(-1) = -$ $\qquad f'(1) = +$ $\qquad f'(3) = -$

rough graph of f

local min at $x = 0$

local max at $x = 2$

Example: The graph of $f\,'(x)$ is shown. Classify the critical numbers for f. In addition, list the intervals of increase and intervals of decrease for f.



It appears that $f'(x)$ exists for all x.

$f'(x) = 0$ iff $x = -2, -1, 1$ ← C.N.

local nothing

rough shape of f

local max at $x = -2$

local min at $x = 1$

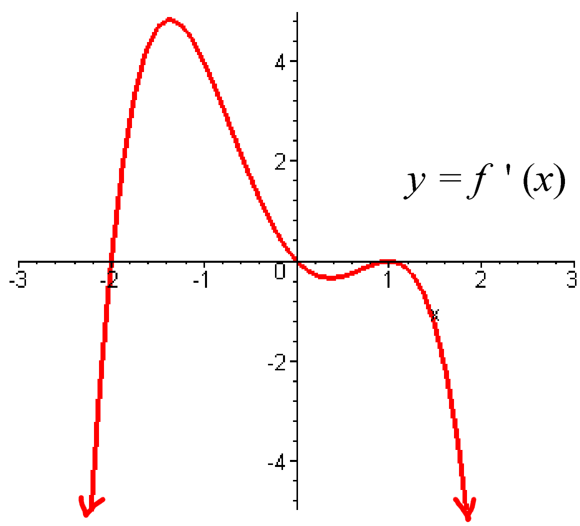
f is increasing on $(-\infty, -2]$ and $[1, \infty)$

f is decreasing on $[-2, 1]$

you can use "round" brackets.

Example: The graph of $f\,'(x)$ is shown. Classify the critical numbers for f. In addition, list the intervals of increase and intervals of decrease for f.

Video



New: How do we determine the absolute extreme values of a function on a closed bounded interval?

Setting: f is a continuous function on $[a, b]$.

1. Find $f(a)$ and $f(b)$.
2. Find $f(c)$ at every c.n. $x = c$ in $[a, b]$
3. Compare.

Example: Find the absolute extreme values for

$$f(x) = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x - 3 \text{ on the interval } [-4, 1].$$

continuous.

1. $f(-4) = \frac{64}{3} + 8 - 24 - 3 = \frac{64}{3} - 19 = \frac{7}{3}$

$f(1) = -\frac{1}{3} + \frac{1}{2} + 6 - 3 = \frac{19}{6}$

2. get c.n. in $[-4, 1]$.

$f'(x) = -x^2 + x + 6$ $\quad f'(x) = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$ $\quad \cancel{x = 3}, \; \underline{\underline{x = -2}}$

$f(-2) = -\frac{31}{3}$

3. Compare

The absolute maximum value is 19/6, and it occurs at $x = 1$. The absolute minimum value is -31/3, and it occurs at $x = -2$.