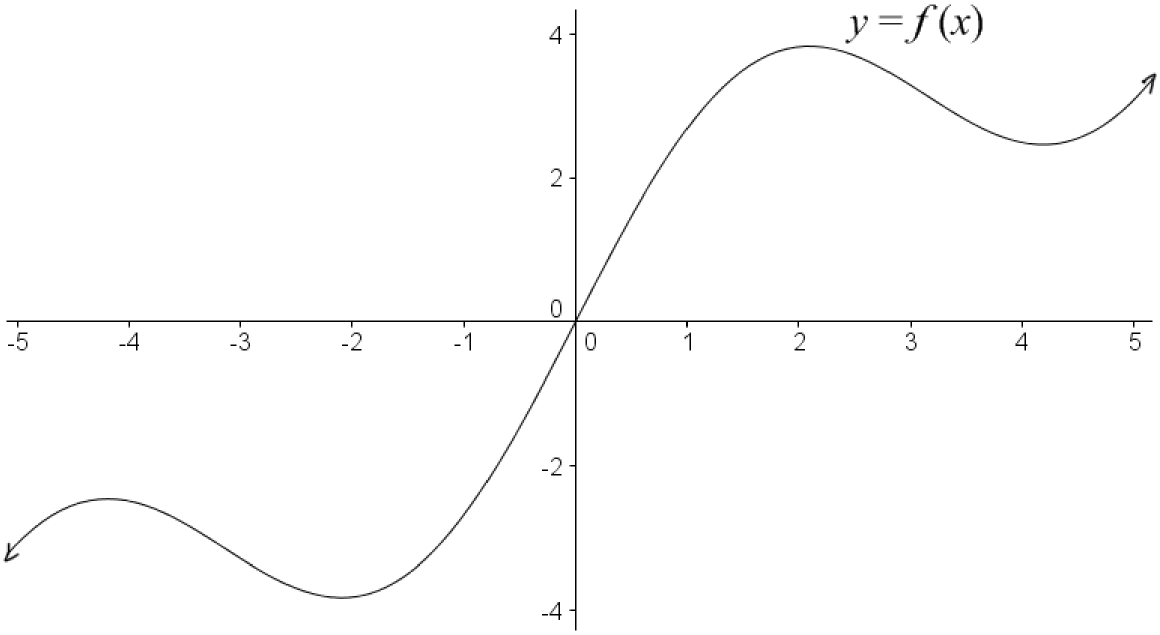
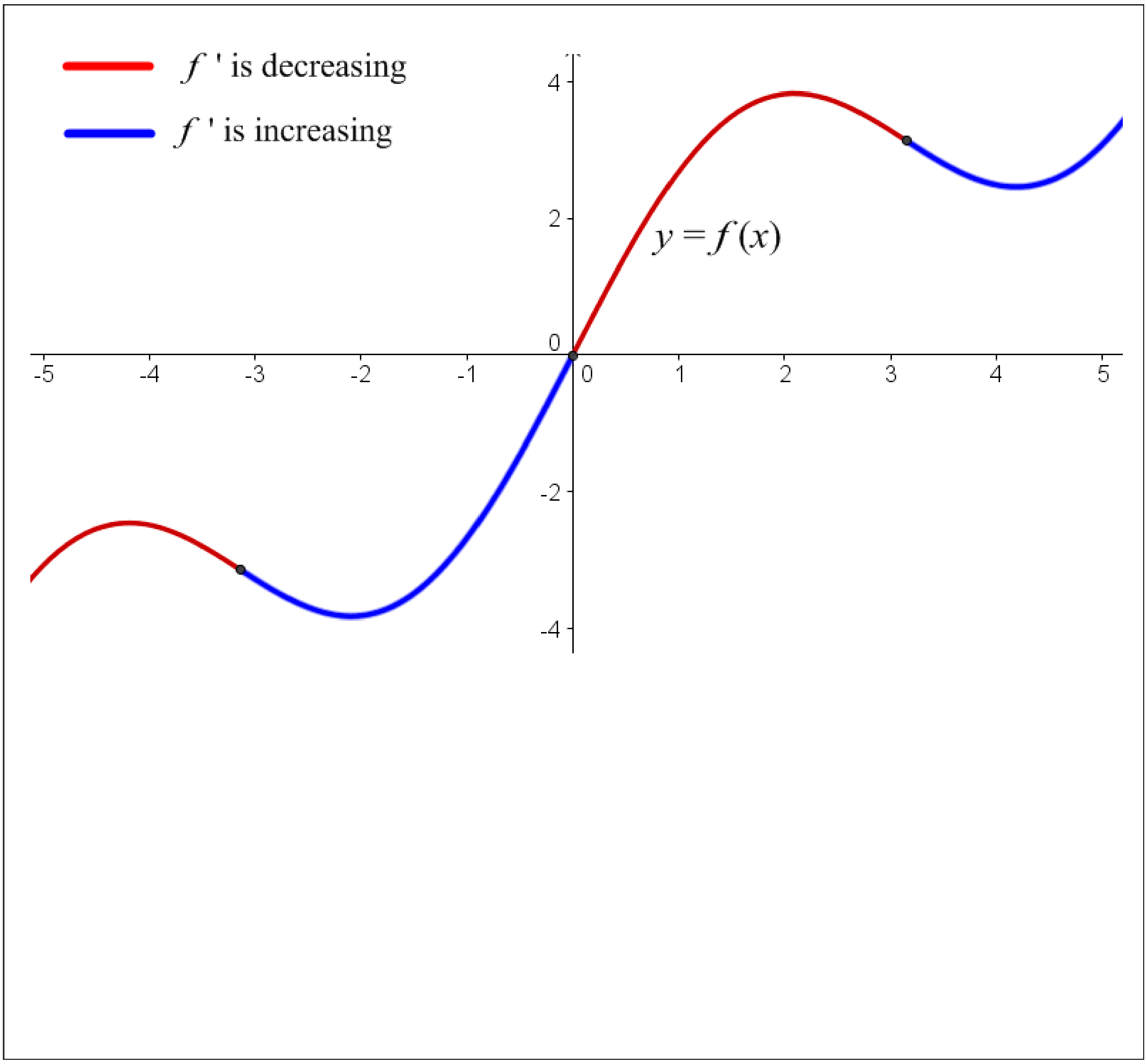
Info...

- **Test 3** is scheduled for November 1-5. The scheduler opens on October 18th.
- **EMCF**s and **Homework** are posted.
- **Test 3 Review** is posted.

Where is $f\,'$ increasing/decreasing?

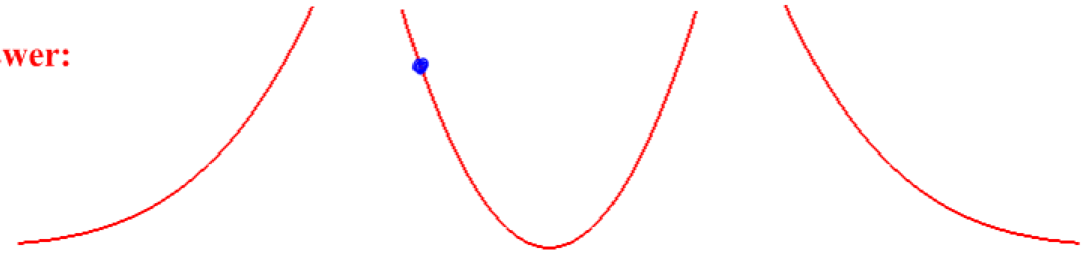




$f\,'$ is decreasing
$f\,'$ is increasing
$y = f(x)$
4
2
0
-2
-4
-5
-4
-3
-2
-1
0
1
2
3
4
5

Question: Suppose $f\,'$ is increasing on an interval. What are the possible shapes for the graph of f over this interval?

Answer:

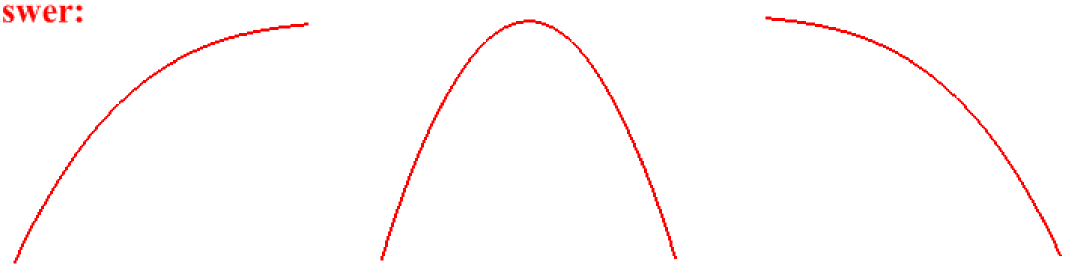


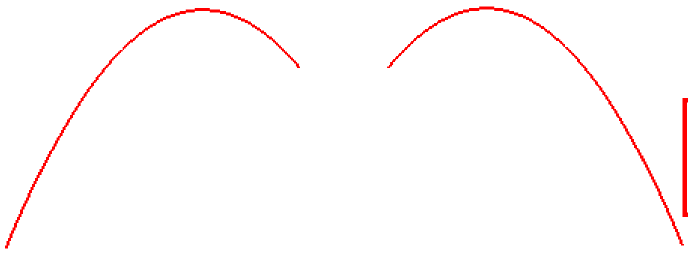
f is in an upward trend

f is **concave up** when $f\,'$ is increasing

Question: Suppose $f\,'$ is decreasing on an interval. What are the possible shapes for the graph of f over this interval?

Answer:



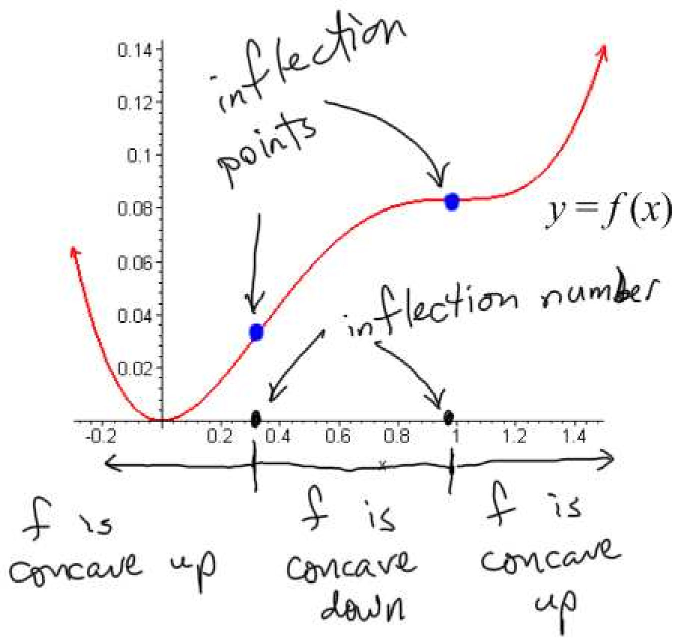


f is in a downward trend

f is **concave down** when $f\,'$ is decreasing

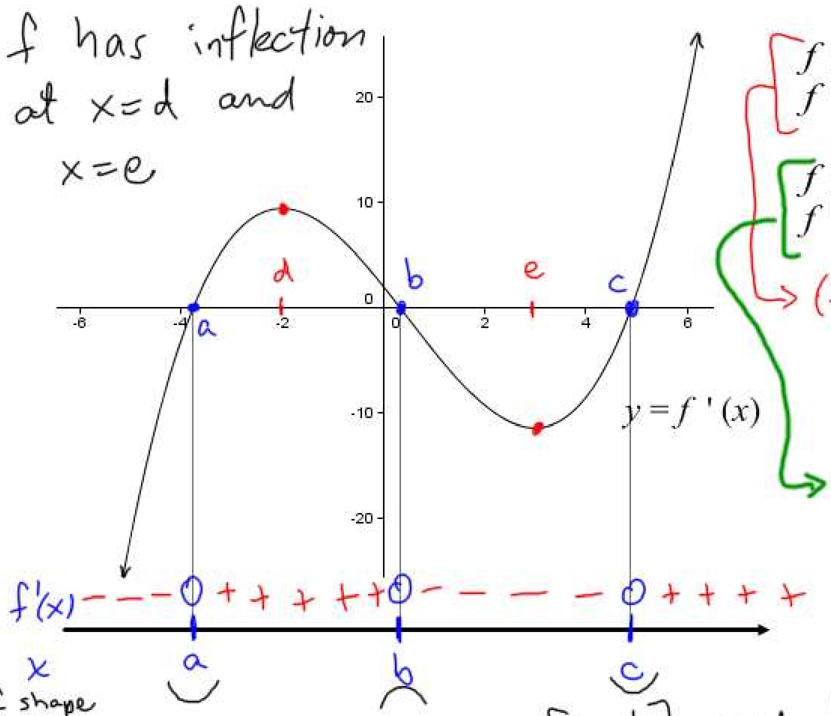
Inflection occurs when Concavity Changes!! ⟷ **and the value of x is in the domain of f**

Example: Identify the inflection points and the intervals of cancavity of the function graphed below.



Example: The graph of $f\,'$ is shown below. List the intervals of increase, decrease, concave up and concave down for f, and classify the critical values for f and list any inflection for f.

f has inflection at $x=d$ and $x=e$



f is concave up when $f\,'$ is increasing. → $(-\infty, d]$ and $[e, \infty)$

f is concave down when $f\,'$ is decreasing. → $[d, e]$

c.n. at $x = a, b, c$

f is increasing on $[a, b]$ and $[c, \infty)$.

f is decreasing on $(-\infty, a]$ and $[b, c]$.

f has local minimums at $x=a$ and $x=c$

f has a local maximum at $x=b$.

on an interval

$\boxed{f\,'\text{ is increasing}}$ if f'' is positive except at finitely many values

$\boxed{f\text{ is c.u.}}$

$\boxed{f\,'\text{ is decreasing}}$ if f'' is negative except at finitely many values

$\boxed{f\text{ is c.d.}}$

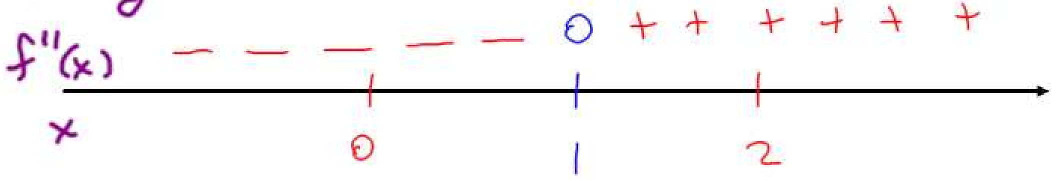
on an interval

Example: Determine the intervals of concavity and the inflection numbers for $f(x) = x^3 - 3x^2 + 2x - 1$

$$f'(x) = 3x^2 - 6x + 2$$

$$f''(x) = 6x - 6 \; . \quad f''(x) = 0 \text{ iff } x = 1$$

concavity chart



$$f''(0) = - \qquad\qquad f''(2) = +$$

f is concave down on $(-\infty, 1]$

f is concave up on $[1, \infty)$

f has inflection at $x = 1$.

The Second Derivative Test
for Classifying Critical Numbers

Suppose $f'(c) = 0$.

Question: What is the expected shape of the graph of f for a local minimum to occur at $x = c$?

$\leftarrow (c, f(c))$

concave up shape

Question: What is the expected shape of the graph of f for a local maximum to occur at $x = c$?

$\swarrow (c, f(c))$

concave down shape

Question: How can the second derivative help us determine the associated shape? Does it ever fail?

second derivative test

$f'(c) = 0$ and $f''(c) > 0$
then f has a local min at $x = c$

$f'(c) = 0$ and $f''(c) < 0$
then f has a local max at $x = c$

$f'(c) = 0$ and $f''(c) = 0$
then we cannot draw any conclusion.

$f(x) = x^3$

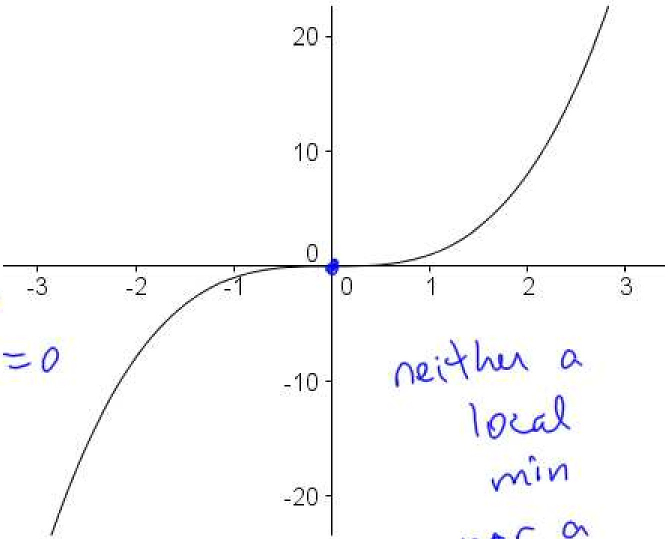
$f'(x) = 3x^2$

c.n. at $x = 0$

b/c $f'(0) = 0$

$f''(x) = 6x$

$\Rightarrow f''(0) = 0$



neither a local min nor a local max

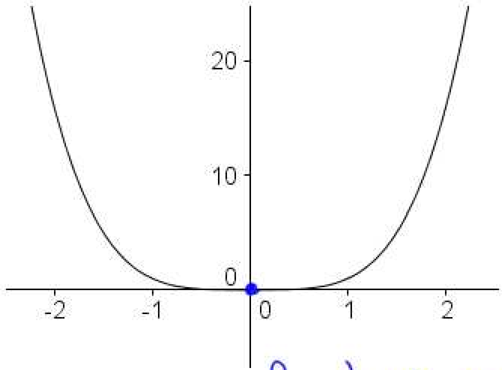
$f(x) = x^4$

$f'(x) = 4x^3$

note: $f'(0) = 0$

$f''(x) = 12x^2$

$f''(0) = 0$



f has a local min at $x=0$

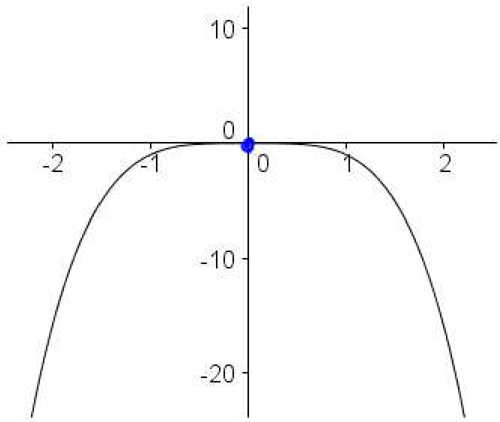
$f(x) = -x^4$

$f'(x) = -4x^3$

$f'(0) = 0$

$f''(x) = -12x^2$

$f''(0) = 0$



f has a local max at $x=0$.

Example: Use the second derivative test to classify the critical numbers of $f(x) = -2x^3 + 3x^2 + 6x + 2$.

$$f'(x) = -6x^2 + 6x + 6$$

$f'(x)$ exists for all x.

$$f'(x) = 0 \iff -6x^2 + 6x + 6 = 0$$

$$x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

c.n. at $x = \frac{1-\sqrt{5}}{2}$ and $x = \frac{1+\sqrt{5}}{2}$

C.U.

$$f''(x) = -12x + 6$$

$$f''\left(\frac{1-\sqrt{5}}{2}\right) = -6 + 6\sqrt{5} + 6 = 6\sqrt{5} > 0$$

$\Rightarrow$ f has a local min at $x = \frac{1-\sqrt{5}}{2}$.

$$f''\left(\frac{1+\sqrt{5}}{2}\right) = -6 - 6\sqrt{5} + 6 = -6\sqrt{5} < 0$$

C.D

$\Rightarrow$ f has a local max at $x = \frac{1+\sqrt{5}}{2}$.