Info

- EMCFs and Homework will be posted for next week.
- Homework is posted and due on Monday.
- A Quiz will be given in lab today.
- An Online Quiz will be due on Monday at 11:59pm.
- Test 3 registration has already started.

The Second Derivative Test
for Classifying Critical Numbers

Suppose $f\,'(c) = 0$ and $f\,''(x)$ exists in an open neighborhood containing $x = c$.

c.n. at $x = c$

If $f\,''(c) > 0$ then f has a local minimum at $x = c$.

If $f\,''(c) < 0$ then f has a local maximum at $x = c$.

If $f\,''(c) = 0$ then anything is possible, and the test fails to give information.

Example: Use the second derivative test to classify the critical numbers of $f(x) = -2x^3 + 6x^2 + 18x + 2$.

$f'(x) = -6x^2 + 12x + 18$
exists for all x.

Set $f'(x) = 0 \iff -6x^2 + 12x + 18 = 0$

$-6(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$\boxed{x = -1, \quad x = 3}$ c.n.

2nd deriv test

$f''(x) = -12x + 12$

$f''(-1) = 24 > 0 \Rightarrow f$ has a local min at $x = -1$

$f''(3) = -24 < 0 \Rightarrow f$ has a local max at $x = 3$

Popper P18

The graph of $y = f\,''(x)$ is shown, and $f\,'(x) = 0$ for $x = -2, 3, 4$.

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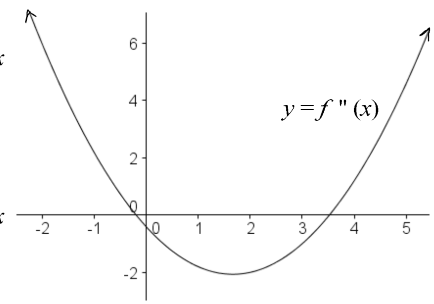


1. Give the smallest value of x where f has a local maximum.

2. Give the largest value of x where f has a local maximum.

3. Give the smallest value of x where f has a local minimum.

4. Give the largest value of x where f has a local minimum.

Example: $\boxed{x=1}$ is a critical number of $f(x) = 2x(x-1)^3 + 3(x-1)^4$. Explain why the second derivative test cannot be used to classify this critical number. Then use the first derivative test to classify the critical number $x = 1$.

$f'(x) = 2x \cdot 3(x-1)^2 + (x-1)^3 \cdot 2 + 12(x-1)^3$

$f'(x) = 6x(x-1)^2 + 14(x-1)^3$

Note: $f'(1) = 0$.

$f''(x) = 6x \cdot 2(x-1) + (x-1)^2 \cdot 6 + 42(x-1)^2$

$= 12x(x-1) + 48(x-1)^2$

$\therefore f''(1) = 0$ Test Fails!

$f'(x) = (x-1)^2 [6x + 14(x-1)]$

$f'(x) = (x-1)^2 [20x - 14]$

set $f'(x) = 0$. solve

C.n. $x = 1$, $x = \frac{14}{20} = \frac{7}{10}$

$f'(x)$ – – – – 0 + + + + + 0 + + + + +

x: 0, $\frac{7}{10}$, $\frac{8}{10}$, 1, 2

$f'(0) = -$ $f'(\frac{8}{10}) = +$ $f'(2) = +$

f shape

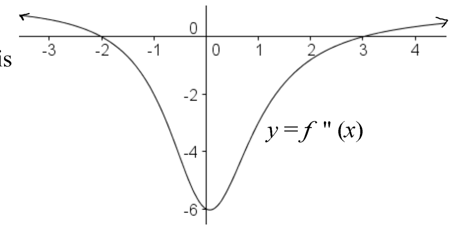
Neither local max nor local min at $x = 1$.

Popper P18

The graph of $y = f''(x)$ is shown.

5. Give the left endpoint of the interval on which the function f is concave down.

6. Give the smallest inflection number for f.

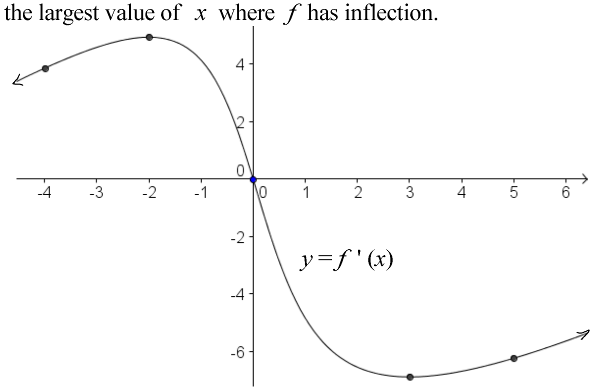


Popper P18

The graph of $y = f'(x)$ is shown.

7. Give the smallest value of x where f has inflection.

8. Give the largest value of x where f has inflection.



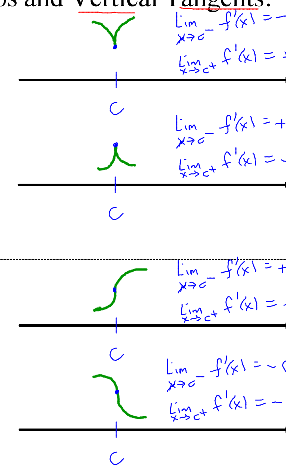
Moving towards graphing...

Identifying Cusps and Vertical Tangents:
Vertical

Cusps:

$f'(c)$ dne

$f(x) = x^{2/3}$



$\lim_{x \to c^-} f'(x) = -\infty$
$\lim_{x \to c^+} f'(x) = +\infty$

$\lim_{x \to c^-} f'(x) = +\infty$
$\lim_{x \to c^+} f'(x) = -\infty$

Vertical Tangents

$f'(c)$ dne

$\lim_{x \to c^-} f'(x) = +\infty$
$\lim_{x \to c^+} f'(x) = +\infty$

$\lim_{x \to c^-} f'(x) = -\infty$
$\lim_{x \to c^+} f'(x) = -\infty$

Determine whether or not the graph of f has a vertical tangent or a vertical cusp at c.

21. $f(x) = (x+3)^{4/3}$; $c = -3$.

Check: $f'(x) = \frac{4}{3}(x+3)^{1/3}$

$f'(-3) = 0$ ← WHAT ??

Neither occurs.

28. $f(x) = 4 - (2-x)^{3/7}$; $c = 2$.

Check: $f'(x) = \frac{3}{7}(2-x)^{-4/7} = \frac{3}{7(2-x)^{4/7}}$

$f'(2)$ dne

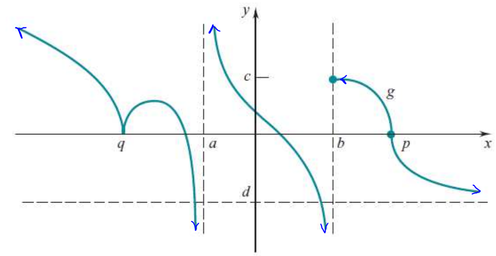
$\lim_{x\to 2^-} f'(x) = \infty$

$\lim_{x\to 2^+} f'(x) = \infty$

f has a vertical tangent at $x = 2$.

Book Exercise:

2. The graph of a function g is given in the figure.



(a) As $x \to \infty, g(x) \to ?$ (b) As $x \to b^+, g(x) \to ?$

(c) Give the equations of the vertical asymptotes, if any.

(d) Give the equations of the horizontal asymptotes, if any.

(e) Give the numbers c, if any, at which the graph of g has a vertical tangent line.

(f) Give the numbers c, if any, at which the graph of g has a vertical cusp.

Moving towards graphing...

Identifying Horizontal and Vertical Asymptotes:

See the video!!

Example: Find the horizontal and vertical asymptotes for

$$f(x) = \frac{8-2x^2}{x^2-2x-8}$$

See the video!!