Information

- Test 3 is 11/01 - 11/05!!
- Practice Test 3 is posted.
- Test 3 covers sections 3.9 - 4.8.
- We will do a partial review today and Friday.

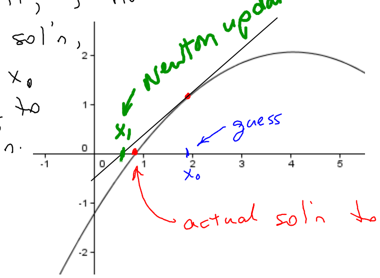
Concept	Comments
1. Newton's Method	Goal: Approximate a solution to $\boxed{f(x) = 0.}$

Need: f twice diff, f' nonzero at sol'n, and x_0 close to sol'n.

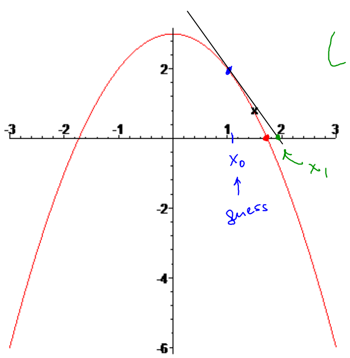
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\vdots$$



Example: The graph of $y = f(x)$ is shown below. Suppose Newton's method is used to approximate a solution to $f(x) = 0$ with one iteration, starting from a guess of x_0. Will the result be larger than or smaller than the actual positive solution? Explain.



LARGER

Example: Use one iteration of Newton's method from a guess of $\boxed{x_0 = 1}$ to approximate a solution to $x^3 - 3x^2 + 2x = 0.1$.

Popper P20

1. $1 + 2 =$

$f(1) = -\frac{1}{10}$

$f'(1) = -1$

$$\underbrace{x^3 - 3x^2 + 2x - \frac{1}{10}}_{f(x)} = 0$$

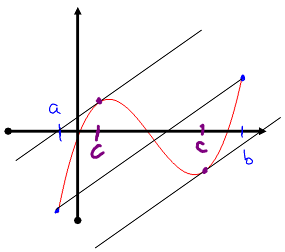
$$f'(x) = 3x^2 - 6x + 2$$

$$x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-1/10}{-1} = 1 - \frac{1}{10}$$

$$= 0.9$$

Concept	Comments
2. MV Theorem	
If f is continuous on $[a,b]$ and differentiable on (a,b) then there is a value c between a and b so that $f'(c)=\frac{f(b)-f(a)}{b-a}$	$f'(c) = \frac{f(b)-f(a)}{b-a}$

at least one



Slope of the T.L. at $x=c$

Slope of the secant line.

Example: Verify the conclusion of the Mean Value Theorem for the function $f(x)=x^2-3x$ on the interval $[-1, 3]$.

$\hookrightarrow f'(x) = 2x-3$

Find c between -1 and 3 so that

$$f'(c) = \frac{f(3)-f(-1)}{3--1}$$

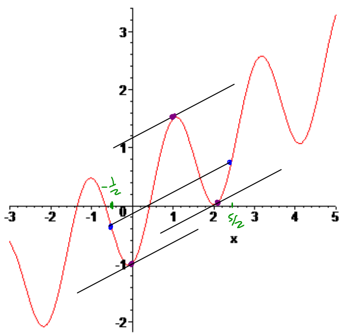
$$2c-3 = \frac{0-4}{4}$$

$$2c-3=-1 \Rightarrow \boxed{c=1}$$

Note: $-1<1<3$.

Example: The graph of $y=f(x)$ is given below. Give the number of values that satisfy the conclusion of the mean value theorem on the interval $[-1/2, 5/2]$.

3



Popper P20

2. $2+2=$

Concept	Comments
3. Differentials	Tangent line approximation
Formula:	The differential of f at x_0 with increment h is $df = f'(x_0)h$

Approximates $f(x_0+h)-f(x_0)$

change in the independent variable

Approximating with differentials:

$$f(x_0+h) - f(x_0) \approx f'(x_0)h$$

$$\underbrace{f(x_0+h)}_{f \text{ where you want the value}} \approx \underbrace{f(x_0) + f'(x_0)h}_{\text{info where you know the value.}}$$

Example: Use differentials to approximate $\sqrt{48.5}$.

Note: $\sqrt{49} = 7.$

$f(x) = \sqrt{x} \qquad f'(x) = \frac{1}{2\sqrt{x}}$

$f(\underbrace{48.5}_{x_0+h}) \approx f(\underbrace{49}_{x_0}) + f'(49)\underbrace{(-0.5)}_{h}$

$= 7 + \frac{1}{14}\left(-\frac{1}{2}\right) = 7 - \frac{1}{28}$

$= \frac{195}{28} = 6.96429$

Note: $\sqrt{48.5} = 6.96419\ldots$

Example: Circular disks are to be created having radius 5 cm, but due to errors in production, the actual radius could vary. The customer wants the total area of the disk to vary by no more than 2 mm^2. Use differentials to estimate the allowable error in the radius.

Popper P20

3. $3 + 2 =$

See the video

Concept	Questions/Comments
4. Increasing A function f is increasing on an interval I if and only if $f(x) < f(y)$ for all x, y in I with $x < y$.	Quick Check: f' exists on I and $f' > 0$ on I except possibly at finitely many places.

Concept	Questions/Comments
5. Decreasing A function f is decreasing on an interval I if and only if $f(x) > f(y)$ for all x, y in I with $x < y$.	Quick Check: f' exists on I and $f' < 0$ on I except possibly at finitely many places.

Example: Give the intervals of increase and decrease for the function $f(x) = 2x^3 - 9x^2 - 24x + 6$.

Popper P20

4. $3 - 5 =$

polynomial

$f'(x) = 6x^2 - 18x - 24$

$f'(x)$ exists for all x

Solve $f'(x) = 0$.

$6x^2 - 18x - 24 = 0$

$6(x^2 - 3x - 4) = 0$

$f'(x) \to 6(x-4)(x+1) = 0$

$\boxed{x = -1,\ x = 4} \leftarrow$ c.n.

Slope chart:

$f'(x)$: $+\ +\ +\ +\ 0\ -\ -\ -\ -\ 0\ +\ +\ +$

x: $-2 \quad -1 \quad 0 \quad 4 \quad 5$

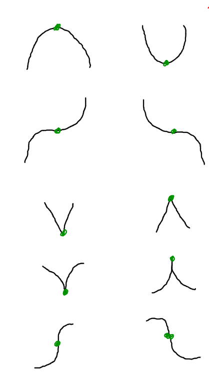
$f'(-2) = +$ $\quad f'(0) = -$ $\quad f'(5) = +$

f is increasing on $(-\infty, -1]$ and $[4, \infty)$.

f is decreasing on $[-1, 4]$.

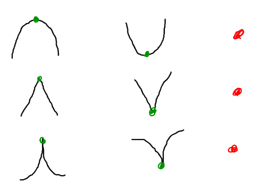
Concept	Questions/Comments
6. Critical Number	Function shapes at a critical number:

A value c is a critical number for a function f if and only if c is in the domain of f and either $f'(c) = 0$ or $f'(c)$ d.n.e.



shape of f near a c.n.

Concept	Questions/Comments
7. Local Max/Min	Graphically:



A function f has a local maximum at a value c if and only if $f(c) \geq f(x)$ for all x near c.

A function f has a local minimum at a value c if and only if $f(c) \leq f(x)$ for all x near c.

Classifying:

1. First derivative test
2. Second derivative test (need f'' to exist)

Example: The function $f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^4 - \frac{1}{3}x^3 + x^2$ has critical numbers at $x = -1$, $x = 0$, $x = 1$, and $x = 2$. Use the first derivative test to classify these critical values.

$f'(x) = x^4 - 2x^3 - x^2 + 2x$

$= x(x^3 - 2x^2 - x + 2)$

$f'(x) = x(x+1)(x-1)(x-2)$

Slope chart:

$f'(x)$: $+\ +\ +\ +\ 0\ -\ -\ 0\ +\ +\ +\ 0\ -\ -\ -\ 0\ +\ +\ +\ +$

x: $-2 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2 \quad \frac{5}{2}$

f shape

$f'(-2) = +$ $\quad f'(-\frac{1}{2}) = -$ $\quad f'(\frac{1}{2}) = +$ $\quad f'(\frac{3}{2}) = -$

$f'(5/4) = +$

f has loc maximums at $x = -1$ and $x = 1$.

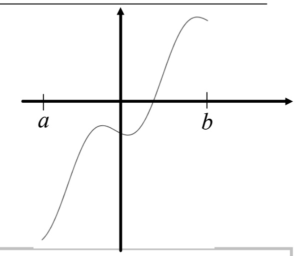
f has loc minimums at $x = 0$ and $x = 2$.

Example: The function $f(x)=\frac{1}{5}x^5-\frac{1}{2}x^4-\frac{1}{3}x^3+x^2$ has critical numbers at $x=-1$, $x=0$, $x=1$, and $x=2$. Use the second derivative test to classify these critical values.

Next Time

Next Time

Concept	Questions/Comments
8. Absolute Max/Min	Graphically:
A function f on an interval I has an absolute maximum at a value c in I if and only if $f(c) \geq f(x)$ for all x in I.	f on $[a,b]$.
A function f on an interval I has an absolute minimum at a value c in I if and only if $f(c) \leq f(x)$ for all x in I.	



Example: Give the maximum value of the function $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2-4x+5$ on the interval $[1,5]$.

Next Time

Example: Find the largest possible value of xy given that x and y are both positive and $2x+y=40$.

Next Time

Example: Find the largest possible area for a rectangle with base on the x-axis and upper vertices on the curve $y = 4 - x^2$.

Next Time

Next Time

Concept	Questions/Comments
9. Concavity	Graphically:
A function f if concave up on an interval I if and only if $f'(x)$ is increasing on I.	
A function f if concave down on an interval I if and only if $f'(x)$ is decreasing on I.	Quick Check:

Next Time

Concept	Questions/Comments
10. Inflection	Graphically:
A function f has inflection at a value c provided c is in the domain of f and the concavity is different on the left of c than it is on the right of c.	Quick Check: ...change in concavity...

Example: The graph of f' is shown below. Use this graph to find classify critical numbers, intervals of increase and decrease, intervals of concavity, and inflection for f. Then give a plausible graph for f.

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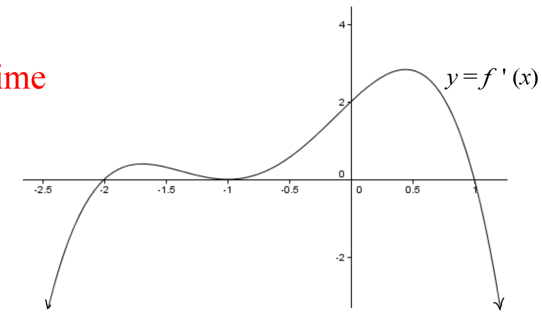

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Concept	Questions/Comments
11. Asymptotes and behavior at the edge of the domain.	Horizontal Asymptotes: Vertical Asymptotes:

Next Time

Concept	Questions/Comments
12. Graphing	

1. Domain
2. Asymptotes and behavior for x near the "edges" of the domain.
3. First Derivative
   - critical numbers
   - slope chart
   - intervals of increase
   - intervals of decrease
   - classify c.n.
4. Second Derivative
   - intervals of concavity
   - inflection
5. Graph it!! (plot plots associated with the information above, along with the y - intercept, and the x - intercept(s) if they are easily found.

Example: Graph $f(x)=\dfrac{x^2}{3-2x}$

Next Time