Information

- Test 3 is 11/01 - 11/05!!
- Practice Test 3 is posted.
- Test 3 covers sections 3.9 - 4.8.
- We will do a partial review today and Friday.

Example: The graph of \( y = f(x) \) is shown below. Suppose Newton's method is used to approximate a solution to \( f(x) = 0 \) with one iteration, starting from a guess of \( x_0 \). Will the result be larger than or smaller than the actual positive solution? Explain.

Example: Use one iteration of Newton's method from a guess of \( x_0 = 1 \) to approximate a solution to \( x^3 - 3x^2 + 2x = 0.1 \).

\[
\begin{align*}
\frac{f(x)}{f'(x)} &= \frac{3x^2 - 6x + 2}{3x - 6} \\
x_1 &= 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-0.1}{-1} = 1 - \frac{1}{10} = 0.9
\end{align*}
\]
Example: Verify the conclusion of the Mean Value Theorem for the function $f(x) = x^2 - 3x$ on the interval $[-1, 3]$.

Let $f'(c) = 2x - 3$.

Find $c$ between $-1$ and $3$ so that

$$f'(c) = \frac{f(3) - f(-1)}{3 - (-1)}$$

$$2c - 3 = \frac{0 - 4}{4}$$

$$2c - 3 = -1$$

$$c = 1$$

Note: $-1 < c < 3$.

Example: The graph of $y = f(x)$ is given below. Give the number of values that satisfy the conclusion of the mean value theorem on the interval $[-1/2, 5/2]$.

Concept | Comments
--- | ---
2. MV Theorem | If $f$ is continuous on $[a,b]$ and differentiable on $(a,b)$ then there is a value $c$ between $a$ and $b$ so that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Slope of the T.L. at $x = c$

Slope of the second line.

3. Differentials | Tangent line approximation

Formula:

The differential of $f$ at $x_0$ with increment $h$ is

$$df = f'(x_0)h$$

The change in the independent variable

Approximating with differentials:

$$f(x_0 + h) \approx f(x_0) + f'(x_0)h$$

$$f(x_0 + h) = f(x_0) + f'(x_0)h$$

$f$ where you want the value.
**Example:** Use differentials to approximate \( \sqrt{48.5} \).

\[
\frac{df}{dx} = \frac{1}{2\sqrt{x}}
\]

\[
f'(49) = \frac{1}{2 \cdot 7}
\]

\[
f(\sqrt{48.5}) \approx f(49) + f'(49)(-0.5)
\]

\[
= 7 + \frac{1}{14} \cdot (-0.5) = 7 - \frac{1}{28}
\]

\[
= \frac{195}{28} = 6.96429
\]

*Note:* \( \sqrt{48.5} = 6.96419 \ldots \)

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**Example:** Circular disks are to be created having radius 5 cm, but due to errors in production, the actual radius could vary. The customer wants the total area of the disk to vary by no more than 2 mm². Use differentials to estimate the allowable error in the radius.

**Quick Check:**

- **Concept:** Increasing
- **Questions/Comments:** A function \( f \) is increasing on an interval \( I \) if and only if \( f(x) < f(y) \) for all \( x, y \) in \( I \) with \( x < y \).

- **Concept:** Decreasing
- **Questions/Comments:** A function \( f \) is decreasing on an interval \( I \) if and only if \( f(x) > f(y) \) for all \( x, y \) in \( I \) with \( x < y \).
Example: Give the intervals of increase and decrease for the function
\[ f(x) = 2x^3 - 9x^2 - 24x + 6. \]

**Pepper P20**

\[ f'(x) = 6x^2 - 18x - 24. \]

\[ f''(x) \text{ exists for all } x. \]

\[ f'(2) = 0. \]
\[ 6(x-3)(x+4) = 0 \]
\[ f'(x) = 6(x-3)(x+4) = 0 \]
\[ x = 3, x = -4 \text{, critical points.} \]

**Step 1:**

\[ f'(x) = \begin{cases} + & x < -4 \text{ or } x > 3 \\ - & -4 < x < 3 \end{cases} \]

\[ f'(x) = - \quad \text{increases on } (-\infty, -4) \text{ and } (3, \infty) \]
\[ f'(x) = + \quad \text{decreases on } [-4, 3] \]

**Concept**

**Questions/Comments**

7. **Local Max/Min**

A function \( f \) has a local maximum at a value \( c \) if and only if \( f(c) \geq f(x) \) for all \( x \) near \( c \).

A function \( f \) has a local minimum at a value \( c \) if and only if \( f(c) \leq f(x) \) for all \( x \) near \( c \).

Graphically:

- \[ \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]
- \[ \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]
- \[ \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]
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- \[ \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]

Classifying:

- 1. **First derivative test**
- 2. **Second derivative test**

Example: The function \( f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - \frac{1}{3}x + 4 \) has critical numbers at \( x = -1, x = 0, x = 1, \) and \( x = 2 \). Use the first derivative test to classify these critical values.

\[ f'(x) = x^2 - 2x^3 - x^2 + 2x \]
\[ = x(x^2 - 2x^2 - x + 2) \]
\[ f'(x) = x(x+1)(x-1)(x-2) \]

**Step 1:**

\[ f'(x) = \begin{cases} + & x < -1 \text{ or } x > 2 \\ - & -1 < x < 1 \text{ or } 1 < x \end{cases} \]

\[ f'(x) = + \quad \text{increases on } (-\infty, -1) \text{ and } (2, \infty) \]
\[ f'(x) = - \quad \text{decreases on } (-1, 1) \text{ and } (1, 2) \]

\[ f \text{ has local maximum at } x = -1 \text{ and } x = 1. \]
\[ f \text{ has local minimum at } x = 0 \text{ and } x = 2. \]
**Example:** The function \( f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^2 - \frac{1}{3}x^3 + x^2 \) has critical numbers at \( x = -1, x = 0, x = 1, \) and \( x = 2. \) Use the second derivative test to classify these critical values.

**Next Time**

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<tr>
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<th>Questions/Comments</th>
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<tbody>
<tr>
<td>8. Absolute Max/Min</td>
<td>A function ( f ) on an interval ( I ) has an absolute maximum at a value ( c ) in ( I ) if and only if ( f(c) \geq f(x) ) for all ( x ) in ( I ).</td>
</tr>
<tr>
<td>Graphically: ( f ) on ([a,b]).</td>
<td><img src="image" alt="Graph of a function" /></td>
</tr>
</tbody>
</table>

A function \( f \) on an interval \( I \) has an absolute minimum at a value \( c \) in \( I \) if and only if \( f(c) \leq f(x) \) for all \( x \) in \( I \).

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**Example:** Give the maximum value of the function \( f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 4x + 5 \) on the interval \([1,5]\).

**Next Time**

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**Example:** Find the largest possible value of \( xy \) given that \( x \) and \( y \) are both positive and \( 2x + y = 40 \).

**Next Time**

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Example: Find the largest possible area for a rectangle with base on the $x$-axis and upper vertices on the curve $y = 4 - x^2$.

Next Time

9. Concavity

A function $f$ is concave up on an interval $I$ if and only if $f''(x)$ is increasing on $I$.

A function $f$ is concave down on an interval $I$ if and only if $f''(x)$ is decreasing on $I$.

Graphically:

Quick Check:

10. Inflection

A function $f$ has inflection at a value $c$ provided $c$ is in the domain of $f$ and the concavity is different on the left of $c$ than it is on the right of $c$.

Graphically:

Quick Check: ...change in concavity...

Example: The graph of $f'$ is shown below. Use this graph to find classify critical numbers, intervals of increase and decrease, intervals of concavity, and inflection for $f$. Then give a plausible graph for $f$.

Next Time
### 11. Asymptotes and behavior at the edge of the domain.

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<td>Horizontal Asymptotes:</td>
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<td>Vertical Asymptotes:</td>
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### 12. Graphing

1. Domain
2. Asymptotes and behavior for $x$ near the "edges" of the domain.
3. First Derivative
   - critical numbers
   - slope chart
   - intervals of increase
   - intervals of decrease
   - classify c.n.
4. Second Derivative
   - intervals of concavity
   - inflection
5. Graph it!! (plot plots associated with the information above, along with the $y$-intercept, and the $x$-intercept(s) if they are easily found.)

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**Example:** Graph $f(x) = \frac{x^2}{3-2x}$