Information

- Test 3 is 11/01 - 11/05!!
- Practice Test 3 is posted.
- Test 3 covers sections 3.9 - 4.8.
- We will do a partial review today and Friday.

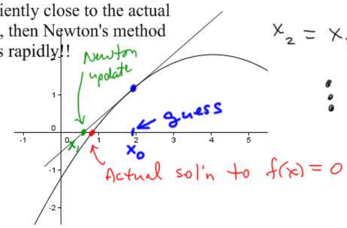
Concept	Comments
1. Newton's Method	Goal: Approximate a solution to $f(x) = 0$.

Suppose f is twice differentiable and the derivative is not zero at the actual solution. If the initial guess is sufficiently close to the actual solution, then Newton's method converts rapidly!!

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

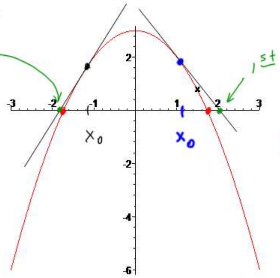
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\vdots$$



Example: The graph of $y = f(x)$ is shown below. Suppose Newton's method is used to approximate a solution to $f(x) = 0$ with one iteration, starting from a guess of x_0. Will the result be larger than or smaller than the actual positive solution? Explain.

1st Newton iterate



1st Newton iterate.

Smaller

LARGER

Example: Use one iteration of Newton's method from a guess of $x_0 = 1$ to approximate a solution to $x^3 - 3x^2 + 2x = 0.1$.

$$\boxed{\begin{array}{l} f(1) = -\frac{1}{10} \\ f'(1) = -1 \end{array}}$$

$$\underbrace{x^3 - 3x^2 + 2x - \frac{1}{10}}_{f(x)} = 0$$

$$f'(x) = 3x^2 - 6x + 2$$

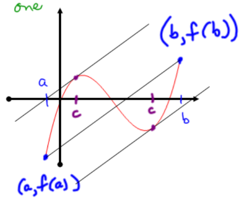
$$x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-\frac{1}{10}}{-1} = 0.9$$

Concept	Comments
2. MV Theorem	$f'(c) = \dfrac{f(b)-f(a)}{b-a}$

If f is continuous on $[a,b]$ and differentiable on (a,b) then there is a value c between a and b so that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

at least one



slope of the tangent line

Slope of the secant line

Example: Verify the conclusion of the Mean Value Theorem for the function $f(x) = x^2 - 3x$ on the interval $[-1, 3]$.

$f'(x) = 2x - 3$

Find c btwn -1 and 3 so that

$$f'(c) = \frac{f(3) - f(-1)}{3 - -1}$$

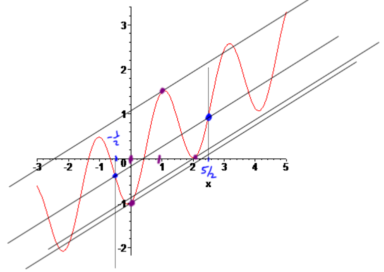
$$2c - 3 = \frac{0 - 4}{4}$$

$$2c - 3 = -1 \Rightarrow 2c = 2 \Rightarrow \boxed{c = 1}$$

Note: $-1 < 1 < 3$.

Example: The graph of $y = f(x)$ is given below. Give the number of values that satisfy the conclusion of the mean value theorem on the interval [-1/2,5/2].

3



Concept	Comments
3. Differentials	← Tangent Line Approx.

The differential of f at x_0 with increment h is given by

Formula: $df = f'(x_0)h$

Here $df \approx f(x_0+h) - f(x_0)$

$f(x_0+h) - f(x_0) \approx f'(x_0)h$

i.e. $f(x_0+h) \approx f(x_0) + f'(x_0)h$

Approximating with differentials:

Example: Use differentials to approximate $\sqrt{48.5}$.

Note: we know $\sqrt{49} = 7$.

$f(x) = \sqrt{x}$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$f(\underset{x_0+h}{48.5}) \approx f(\underset{x_0}{49}) + f'(\underset{x_0}{49})(\underset{h}{-.5})$

$= 7 + \frac{1}{14}\left(-\frac{1}{2}\right)$

$= 7 - \frac{1}{28} = \frac{195}{28} = 6.9642857$

Also $\sqrt{48.5} = 6.9641941\ldots$

Example: Circular disks are to be created having radius 5 cm, but due to errors in production, the actual radius could vary. The customer wants the total area of the disk to vary by no more than 2 mm^2. Use differentials to estimate the allowable error in the radius.

5 cm = 50 mm

$A = \pi r^2$

$dA = 2\pi r \cdot h$

Approx the error in area

$\underbrace{\text{increment in radius}}_{\text{error in radius}}$

$2 = 100\pi h \Rightarrow h = \frac{1}{50\pi}$ mm

$= 0.006366$ mm

Concept	Questions/Comments
4. Increasing A function f is increasing on an interval I if and only if $f(x) < f(y)$ for all x, y in I with $x < y$.	Quick Check: f' exists on I and $f' > 0$ on I except possibly at finitely many places.

Concept	Questions/Comments
5. Decreasing A function f is decreasing on an interval I if and only if $f(x) > f(y)$ for all x, y in I with $x < y$.	Quick Check: f' exists on I and $f' < 0$ on I except possibly at finitely many places.

Example: Give the intervals of increase and decrease for the function
$f(x) = 2x^3 - 9x^2 - 24x + 6.$

← polynomial

$f'(x) = 6x^2 - 18x - 24 = 6(x-4)(x+1)$

$f'(x)$ exists for all x.

$f'(x) = 0 \iff 6x^2 - 18x - 24 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = -1 \text{ or } x = 4}$

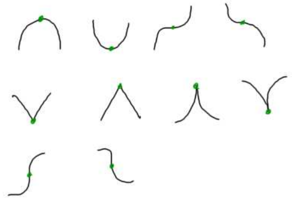
Slope chart

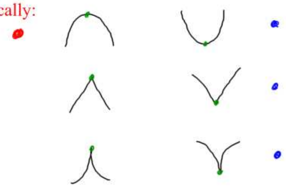
$f'(x)$	+ + + + +	0	− − − − − −	0	+ + + +		
x	-2	-1	0	4	5		

$f'(-2) = +$ $f'(0) = -$ $f'(5) = +$

f is increasing on $(-\infty, -1]$ and $[4, \infty)$.

f is decreasing on $[-1, 4]$.

Concept	Questions/Comments
6. Critical Number	Function shapes at a critical number:
A value c is a critical number for a function f if and only if c is in the domain of f and either $f'(c) = 0$ or $f'(c)$ d.n.e.	

Concept	Questions/Comments
7. Local Max/Min	Graphically: 
A function f has a local maximum at a value c if and only if $f(c) \geq f(x)$ for all x near c.	
A function f has a local minimum at a value c if and only if $f(c) \leq f(x)$ for all x near c.	Classifying: • 1^{st} derivative test • 2^{nd} derivative test (can fail)

Example: The function $f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^4 - \frac{1}{3}x^3 + x^2$ has critical numbers at $x = -1$, $x = 0$, $x = 1$, and $x = 2$. Use the first derivative test to classify these critical values.

polynomial

$f'(x) = x^4 - 2x^3 - x^2 + 2x$

$= x(x^3 - 2x^2 - x + 2)$

$f'(x) = x(x+1)(x-1)(x-2)$

Slope chart:

$f'(x)$: + + + + 0 − − − − 0 + + + 0 − − − 0 + + + +

x: $-\frac{3}{2}$, -1, $-\frac{1}{2}$, 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, $\frac{5}{2}$

f shape: ∩ ∪ ∩ ∪

$f'(-\frac{3}{2}) = +$ $f'(-\frac{1}{2}) = -$ $f'(\frac{1}{2}) = +$ $f'(\frac{3}{2}) = -$

$f'(5/2) = +$

From the first derivative test,
f has local maximums at $x=-1$ and $x=1$,
and
f has local minimums at $x=0$ and $x=2$.

Example: The function $f(x)=\frac{1}{5}x^5-\frac{1}{2}x^4-\frac{1}{3}x^3+x^2$ has critical numbers at $x=-1$, $x=0$, $x=1$, and $x=2$. Use the second derivative test to classify these critical values.

$f'(x) = x^4 - 2x^3 - x^2 + 2x$

$f''(x) = 4x^3 - 6x^2 - 2x + 2$

$f''(-1) = -4 - 6 + 2 + 2 = -6 < 0$

$\therefore$ f has a local max at $x=-1$.

$f''(0) = 2 > 0$ $\therefore$ f has a local min at $x=0$.

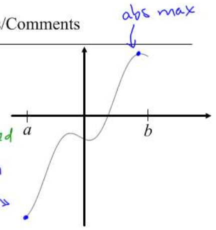
$f''(1) = 4 - 6 - 2 + 2 = -2 < 0$

$\therefore$ f has a local max at $x=1$.

$f''(2) = 32 - 24 - 4 + 2 > 0$

$\therefore$ f has a local min at $x=2$.

Concept	Questions/Comments
8. Absolute Max/Min	Graphically:
A function f on an interval I has an absolute maximum at a value c in I if and only if $f(c) \geq f(x)$ for all x in I.	f on $[a,b]$. closed bounded interval
A function f on an interval I has an absolute minimum at a value c in I if and only if $f(c) \leq f(x)$ for all x in I.	Quick Check: 1. Find $f(a)$ and $f(b)$. 2. Find all critical values in the interval $[a,b]$, and evaluate f at each of these. 3. Compare the values from 1 and 2.



Example: Give the maximum value of the function $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2-4x+5$ on the interval $[1,5]$.

1. $f(1) = \frac{1}{3} - \frac{3}{2} - 4 + 5 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

$f(5) = \frac{125}{3} - \frac{75}{2} - 20 + 5$

$= \frac{125}{3} - \frac{75}{2} - 15 = \frac{125}{3} - \frac{105}{2}$

$= \frac{250 - 315}{6} = \frac{-65}{6}$

2. $f'(x) = x^2 - 3x - 4 = (x-4)(x+1)$

exists for all x.

Set $f'(x)=0$. $(x-4)(x+1)=0$

$\cancel{x=-1}$ and $x=4$

$f(4) = \frac{64}{3} - 24 - 16 + 5 = \frac{64}{3} - 35 = \frac{64-105}{3}$

$= -\frac{41}{3}$

3. Compare.

The abs min value is $\frac{-41}{3}$, and it occurs at $x=4$.

The abs max value is $-\frac{1}{6}$, and it occurs at $x=1$.

Example: Find the largest possible value of xy given that x and y are both positive and $2x+y=40$.

Next Time

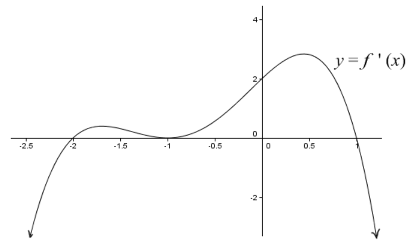
Example: Find the largest possible area for a rectangle with base on the x-axis and upper vertices on the curve $y = 4 - x^2$.

Next Time

Concept	Questions/Comments
9. Concavity	Graphically:
A function f if concave up on an interval I if and only if $f'(x)$ is increasing on I.	
A function f if concave down on an interval I if and only if $f'(x)$ is decreasing on I.	Quick Check:
	Next Time

Concept	Questions/Comments
10. Inflection	Graphically:
A function f has inflection at a value c provided c is in the domain of f and the concavity is different on the left of c than it is on the right of c.	
	Quick Check: ...change in concavity...
	Next Time

Example: The graph of $f\,'$ is shown below. Use this graph to find classify critical numbers, intervals of increase and decrease, intervals of concavity, and inflection for f. Then give a plausible graph for f.



Next Time

Concept	Questions/Comments
11. Asymptotes and behavior at the edge of the domain.	Horizontal Asymptotes: Vertical Asymptotes:

Next Time

Concept	Questions/Comments
12. Graphing	**Next Time**

1. Domain
2. Asymptotes and behavior for x near the "edges" of the domain.
3. First Derivative
   - critical numbers
   - slope chart
   - intervals of increase
   - intervals of decrease
   - classify c.n.
4. Second Derivative
   - intervals of concavity
   - inflection
5. Graph it!! (plot plots associated with the information above, along with the y - intercept, and the x - intercept(s) if they are easily found.

Example: Graph $f(x) = \dfrac{x^2}{3-2x}$

Next Time