Information

- Test 3 is 11/01 - 11/05!!
- Practice Test 3 is posted.
- Test 3 covers sections 3.9 - 4.8.
- We will do another portion of the review today.

Example: The function $f(x)=\frac{1}{5}x^5-\frac{1}{2}x^4-\frac{1}{3}x^3+x^2$ has critical numbers at $x=-1$, $x=0$, $x=1$, and $x=2$. Use the second derivative test to classify these critical values.

Popper P21

1. $1+2=$

$f'(x) = x^4 - 2x^3 - x^2 + 2x$

c.n. $x = -1, 0, 1, 2$

$f''(x) = 4x^3 - 6x^2 - 2x + 2$

$f''(-1) = -4 - 6 + 2 + 2 = -6 < 0$

f has a local max at $x=-1$.

$f''(0) = 2 > 0$

f has a local min at $x=0$.

$f''(1) = 4 - 6 - 2 + 2 = -2 < 0$

f has a local max at $x=1$.

$f''(2) = 32 - 24 - 4 + 2 > 0$

f has a local min at $x=0$.

Concept	Questions/Comments

8. Absolute Max/Min

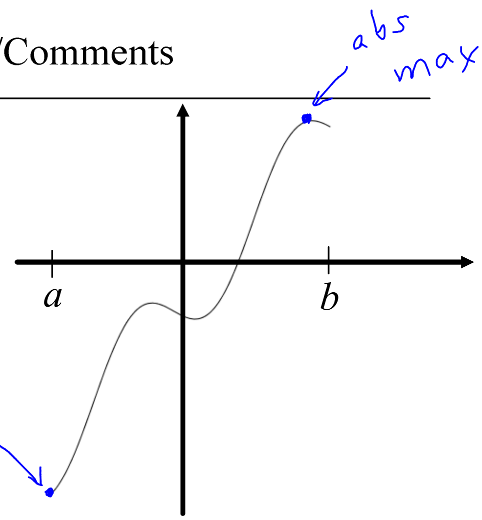
A function f on an interval I has an absolute maximum at a value c in I if and only if $f(c) \geq f(x)$ for all x in I.

A function f on an interval I has an absolute minimum at a value c in I if and only if $f(c) \leq f(x)$ for all x in I.

Graphically:

f on $[a,b]$.

abs min



Quick Check:

1. Find $f(a)$ and $f(b)$.

2. Find all critical values in the interval $[a,b]$, and evaluate f at each of these.

3. Compare the values from 1 and 2.

Example: Give the maximum value of the function $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2-4x+5$ on the interval $[1,5]$.

Popper P21

2. 1 – 3 =

1. $f(1) = -\frac{1}{6}$

$f(5) = -\frac{65}{6}$

2. Find c.n. in $[1,5]$

$f'(x) = x^2 - 3x - 4$ ← exists for all x.

Set $f'(x) = 0 \iff x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$.

$f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2-4x+5$

$\cancel{x=-1}$, $x = 4$ c.n.

$f(4) = -\frac{41}{3}$

3. Compare

∴ The abs max value of f is $-\frac{1}{6}$, and it occurs at $x = 1$.

Example: Find the largest possible value of xy given that x and y are both positive and $2x + y = 40$.

maximize

$$M = xy$$

where $\boxed{2x + y = 40}$

and $x, y > 0$.

$y = 40 - 2x$

$$M(x) = x(40 - 2x), \quad 0 < x < 20$$

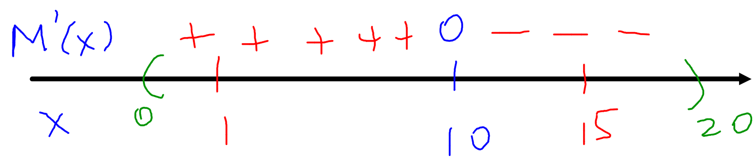
not a closed bound interval, use slope chart

$$M(x) = 40x - 2x^2$$

$$M'(x) = 40 - 4x$$

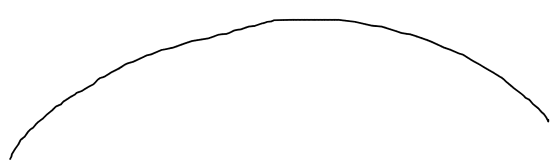
$M'(x) = 0$ when $x = 10$

slope chart



$M'(1) = +$ $\quad$ $M'(15) = -$

Shape of f



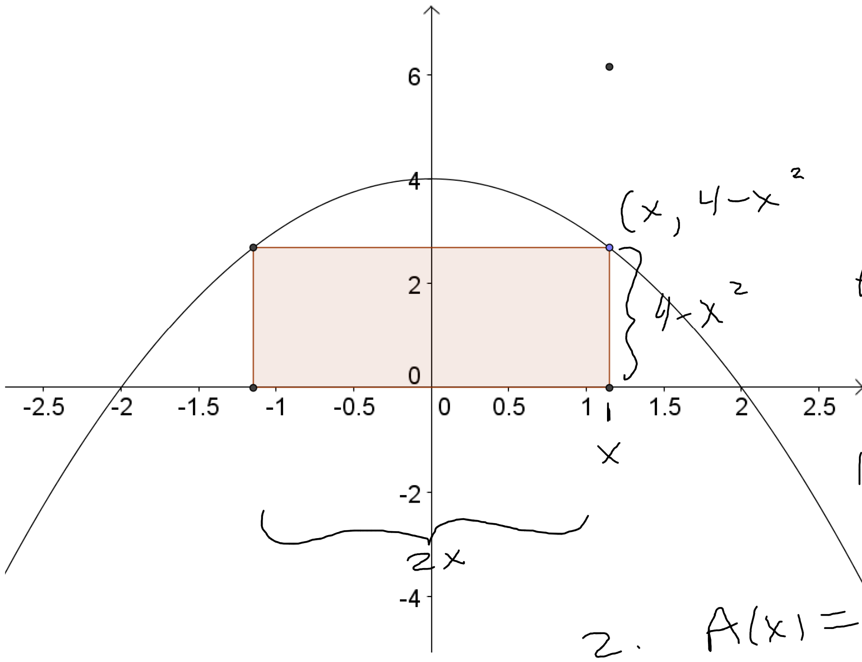
$\therefore$ $M(x)$ has an abs. max. at $x = 10$.

The largest possible value is $\underline{\underline{M(10) = 200}}$

Example: Find the largest possible area for a rectangle with base on the x-axis and upper vertices on the curve $y = 4 - x^2$.

Popper P21

3. $7 - 3 =$



maximize

$A(x) = 2x(4-x^2)$

$0 \leq x \leq 2$

1. $A(0) = 0$

$A(2) = 0$

2. $A(x) = 8x - 2x^3$

$A'(x) = 8 - 6x^2$

$A'(x) = 0 \iff 8 - 6x^2 = 0$

$x = \cancel{\pm}\frac{2}{\sqrt{3}}$

$A(x) = 2x(4-x^2)$

$A\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}}\left(4 - \frac{4}{3}\right)$

$= \frac{32}{3\sqrt{3}}$

3. Compare.

Max Area is $\frac{32}{3\sqrt{3}} \approx 6.1580\ldots$

Concept	Questions/Comments

9. Concavity

A function f if concave up on an interval I if and only if $f'(x)$ is increasing on I.

A function f if concave down on an interval I if and only if $f'(x)$ is decreasing on I.

Graphically: Shape of f

C.U.

C.D.

Quick Check:

- f is C.U. on an interval I if $f''(x) > 0$ at all but finitely many values x in I.
- f is C.D. on an interval I if $f''(x) < 0$ at all but finitely many values x in I.

Concept	Questions/Comments
10. Inflection A function f has inflection at a value c provided c is in the domain of f and the concavity is different on the left of c than it is on the right of c.	Graphically: Quick Check: ...change in concavity...

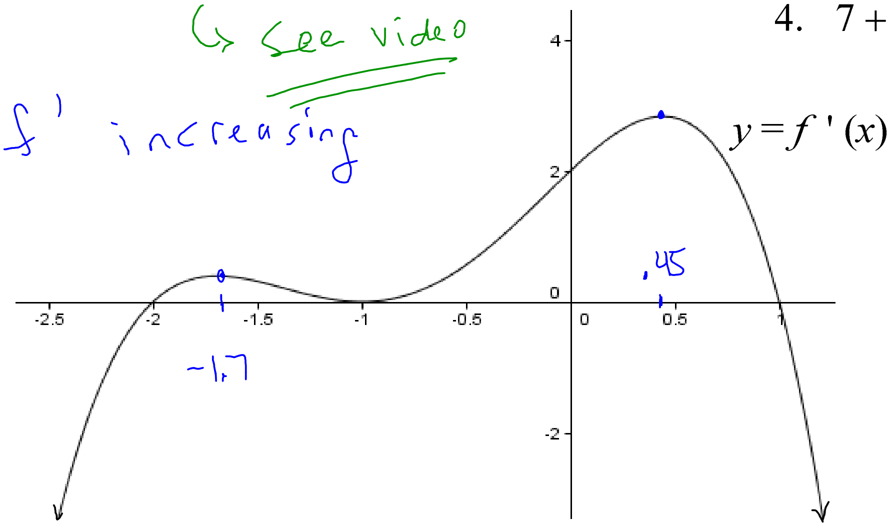
Example: The graph of $f\,'$ is shown below. Use this graph to classify critical numbers, intervals of increase and decrease, intervals of concavity, and inflection for f. Then give a plausible graph for f.

Popper P21

4. $7 + 3 =$

↳ See video

C.U.: f' increasing



C.n.: $x = -2, -1, 1$

f is increasing on $[-2, 1]$

f is decreasing on $(-\infty, -2]$ and $[1, \infty)$

f is C.U. on $(-\infty, -1.7]$ and $[-1, .45]$

f is C.D. on $[-1.7, -1]$ and $[.45, \infty)$

Concept	Questions/Comments
11. Asymptotes and behavior at the edge of the domain.	Horizontal Asymptotes: Vertical Asymptotes:

Concept	Questions/Comments
12. Graphing	

1. Domain
2. Asymptotes and behavior for x near the "edges" of the domain.
3. First Derivative
    - critical numbers
    - slope chart
    - intervals of increase
    - intervals of decrease
    - classify c.n.
4. Second Derivative
    - intervals of concavity
    - inflection
5. Graph it!! (plot plots associated with the information above, along with the y - intercept, and the x - intercept(s) if they are easily found.

Example: Graph $f(x) = \dfrac{x^2}{3-2x}$