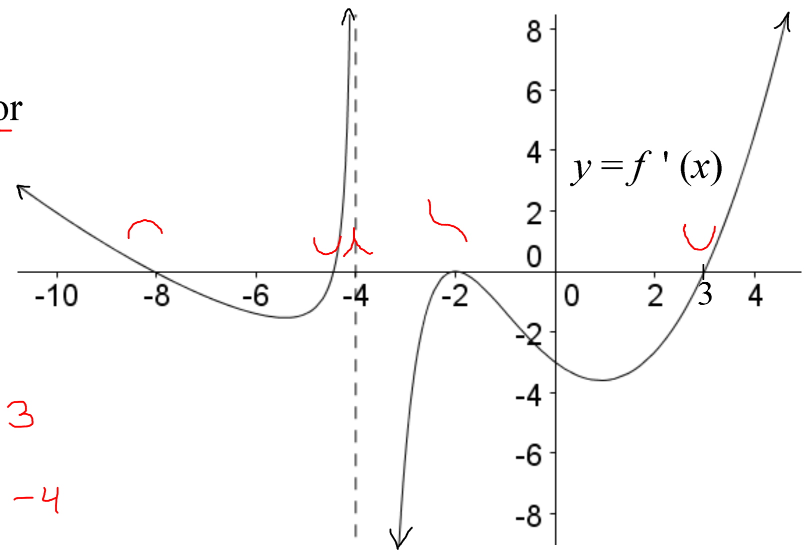
Information

- Test 3 is 11/01 - 11/05!!
- Practice Test 3 is posted.
- Test 3 covers sections 3.9 - 4.8.
- We will do another portion of the review today.
- There will be an online live review Tuesday night from 8:15 to 10:15.

Popper P22

The function f is continuous for all values of x. The graph of $y = f\,'(x)$ is shown.



1. Give the number of critical numbers of f. 5
2. Give the largest value of x where f has a local minimum. 3
3. Give the largest value of x where f has a local maximum. −4
4. Give the number of inflection numbers for f. 3
5. Give the number of intervals of increase for f. 3
6. Give the number of local maximums for f. 2
7. Give the number of intervals of concave up for f.

3

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Concept	Questions/Comments
11. Asymptotes and behavior at the edge of the domain.	Horizontal Asymptotes: $y = c$ is a H.A. iff either $\lim_{x \to -\infty} f(x) = c$ or $\lim_{x \to \infty} f(x) = c$
	Vertical Asymptotes: $\to$ infinite discont.

Concept	Questions/Comments
12. Graphing	

1. Domain
2. Asymptotes and behavior for x near the "edges" of the domain.
3. First Derivative
    - critical numbers
    - slope chart
    - intervals of increase
    - intervals of decrease
    - classify c.n.
4. Second Derivative
    - intervals of concavity
    - inflection
5. Graph it!! (plot plots associated with the information above, along with the y - intercept, and the x - intercept(s) if they are easily found.

Example: Graph $f(x)=\dfrac{x^2}{3-2x}$ ← rational function.

1. Domain: All x except $x=\frac{3}{2}$.

   b/c the denominator is only zero at $x=3/2$.

2. Horizontal Asympt: None

Edge:

$$\lim_{x\to\infty}\frac{x^2}{3-2x}=\lim_{x\to\infty}\frac{x^{\cancel{2}}}{\cancel{x}\left(\frac{3}{x}-2\right)}$$

$$=\lim_{x\to\infty}\frac{x}{\frac{3}{x}-2}=-\infty$$

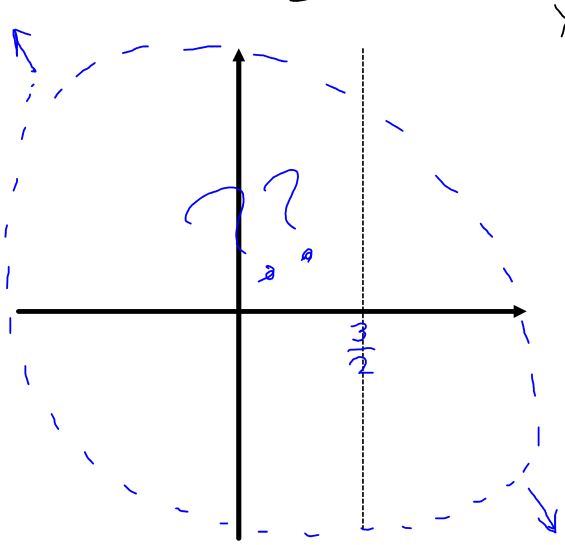
$\frac{3}{x}\to 0$

$$\lim_{x\to-\infty}\frac{x^2}{3-2x}=\cdots=\lim_{x\to-\infty}\frac{x}{\frac{3}{x}-2}=\infty$$

$\frac{3}{x}\to 0$

Vertical Asympt:

$x=\frac{3}{2}$.



$$f(x) = \frac{x^2}{3-2x}$$

3. $$f'(x) = \frac{(3-2x)2x - x^2(-2)}{(3-2x)^2}$$

$$= \frac{6x - 4x^2 + 2x^2}{(3-2x)^2} = \frac{6x - 2x^2}{(3-2x)^2}$$

Note: $f'(x)$ exists for all x except $x = \frac{3}{2}$, and $\frac{3}{2}$ is not in the domain of f.

$$f'(x) = 0 \quad \text{iff} \quad \frac{6x - 2x^2}{(3-2x)^2} = 0$$

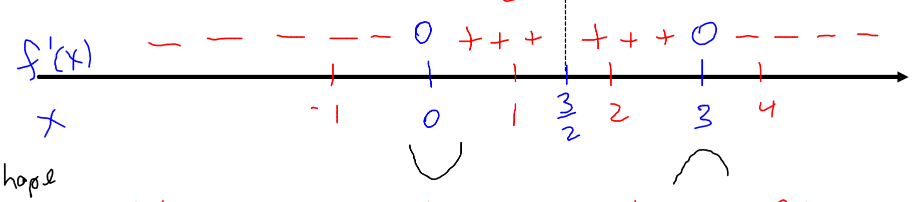
$$f'(x) = \frac{6x - 2x^2}{(3-2x)^2}$$

$$6x - 2x^2 = 0$$

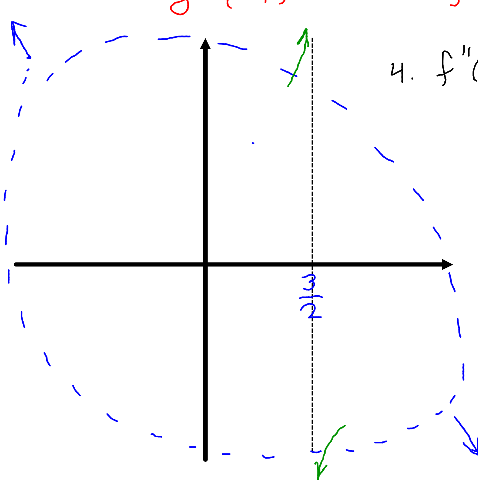
$$2x(3 - x) = 0$$

$$\boxed{x = 0,\ x = 3}$$ C.n.

Slope chart



$f'(-1) = -$ $\quad f'(1) = +$ $\quad f'(2) = +$ $\quad f'(4) = -$



4. $$f''(x) = \frac{(3-2x)^{\cancel{2}}(6-4x) - (6x-2x^2)2\cancel{(3-2x)}(-2)}{(3-2x)^{\cancel{4}\,3}}$$

$$= \frac{(3-2x)(6-4x) + 4(6x - 2x^2)}{(3-2x)^3}$$

$$= \frac{18 - \cancel{12x} - \cancel{12x} + \cancel{8x^2} + \cancel{24x} - \cancel{8x^2}}{(3-2x)^3}$$

$$f''(x) = \frac{18}{(3-2x)^3}$$

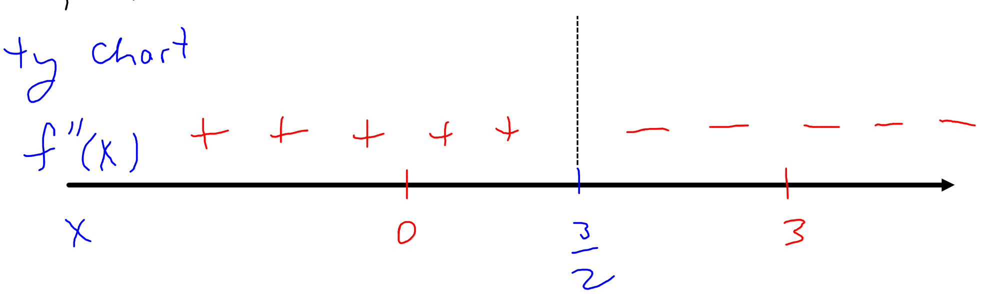
Exists at every x except $x = 3/2$.

$$f''(x) = \frac{18}{(3-2x)^3}$$

Exists at every x
except $x = 3/2$.

Note: $f''(x)$ is never 0.

Concavity chart



$f''(0) = +$ $f''(3) = -$

f is C.U. for $x < \frac{3}{2}$

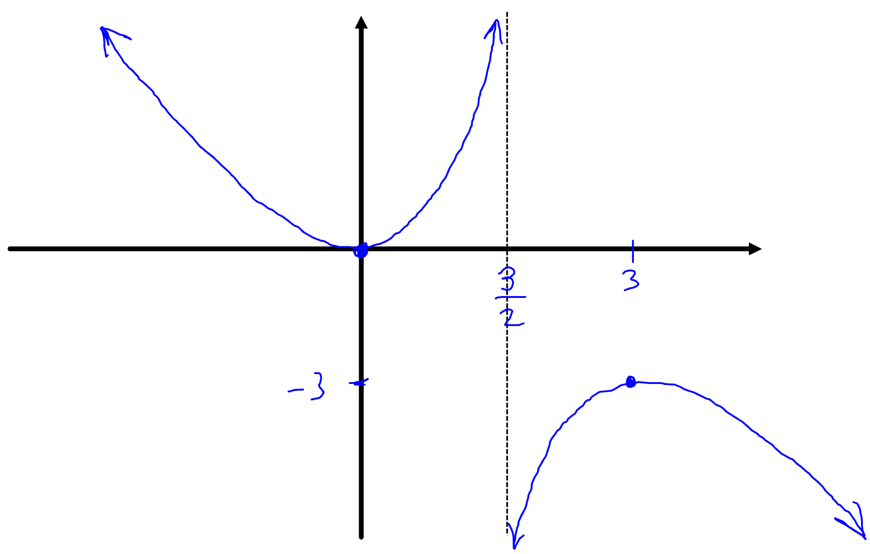
f is C.D. for $x > 3/2$.

Graph: Plot c.p.

$$f(x) = \frac{x^2}{3-2x}$$

$(0, f(0)) = (0, 0)$ ∪

$(3, f(3)) = (3, -3)$ ∩



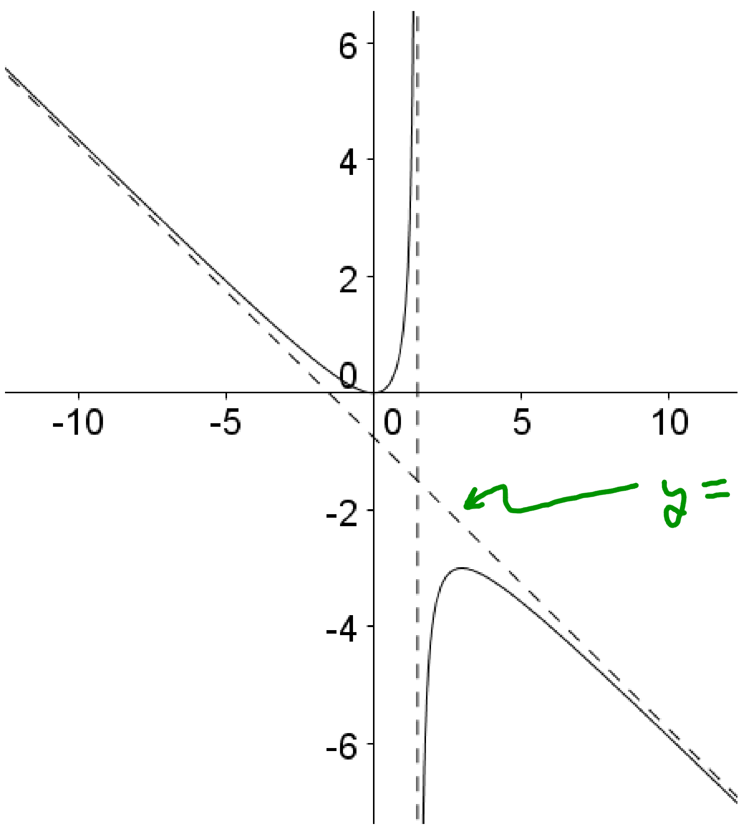
Let's see if we can pick up the oblique asymptote(s).

$$f(x) = \frac{x^2}{3-2x} = \frac{x^2}{-2x+3} = \underbrace{-\tfrac{1}{2}x - \tfrac{3}{4}}_{\text{Line}} + \frac{9/4}{-2x+3}$$

$\rightarrow 0$ as $x \rightarrow \pm\infty$

Oblique

$$\begin{array}{r|l} & -\tfrac{1}{2}x - \tfrac{3}{4} \\ \hline -2x+3 & x^2 + 0x + 0 \\ & -(x^2 - \tfrac{3}{2}x) \\ \hline & \tfrac{3}{2}x + 0 \\ & -(\tfrac{3}{2}x - \tfrac{9}{4}) \\ \hline & \tfrac{9}{4} \end{array}$$



$y = -\frac{1}{2}x - \frac{3}{4}$