Net Area and Integrals

- We are in chapter 5.
- New EMCFs and Homework will be posted.
- Test 3 runs through Monday.
- You have an online quiz and a practice test due Monday.
- $\boxed{\text{No office hours today!!}}$

Review: The first part of the **Fundamental Theorem of Calculus**

Theorem: If f is a continuous function on the interval $[a,b]$, and $\boxed{F \text{ is an anti derivative of } f,}$ then $F'(x) = f(x)$

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

integrand

integral sign

Riemann Integral of $f(x)$ on $[a,b]$.

Limits of integration.

Example: **Compute** $\int_{-1}^{2}(2x^2 - 3x - 5)\,dx$. $= \left(\frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x\right)\Big|_{-1}^{2}$

continuous on $[-1,2]$.

We need $F(x)$ so that

$F'(x) = 2x^2 - 3x - 5$

$F(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x$

$= \left(\frac{16}{3} - 6 - 10\right) - \left(-\frac{2}{3} - \frac{3}{2} + 5\right)$

$= -21 + \frac{3}{2} + \frac{18}{3}$

$= -15 + \frac{3}{2}$

$= -\frac{27}{2}$.

P23

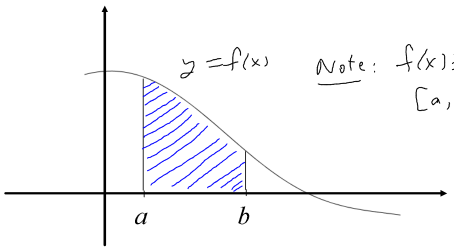
1. $\int_1^2 (x^2 - 3x)\,dx =$

2. $\int_0^{\pi/2} \cos(x)\,dx =$

Review:

The **Riemann** Integral of f from a to b: $\boxed{\int_a^b f(x)\,dx}$ = Area

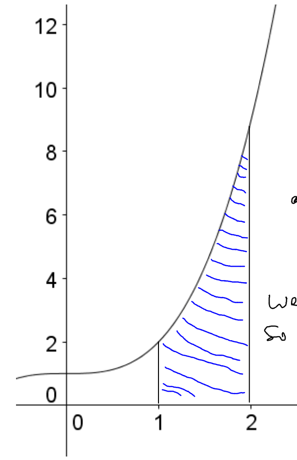
1. What do we get if f is nonnegative on the interval $[a,b]$?



Note: $f(x) \geq 0$ on $[a,b]$.

i.e. If $f(x) \geq 0$ on $[a,b]$ then $\int_a^b f(x)\,dx$ = the area bounded between $y=f(x)$ and the x-axis for x between a and b.

Example: Give the area bounded between the x-axis and the graph of $y = x^3 + 1$ on the interval $[1,2]$.



Note: $f(x) = x^3 + 1$ is nonnegative on $[1,2]$.

$\therefore$ Area $= \int_1^2 (x^3+1)\,dx$

We need $F(x)$ so that $F'(x) = x^3 + 1$

$F(x) = \frac{1}{4}x^4 + x$

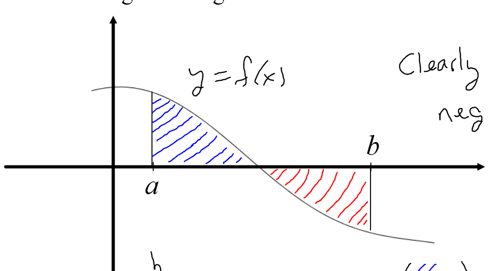
$= \left(\frac{1}{4}x^4 + x\right)\Big|_1^2$

$= (4+2) - \left(\frac{1}{4} + 1\right)$

$= 5 - \frac{1}{4} = \frac{19}{4}$.

The **Riemann** Integral of f from a to b: $\int_a^b f(x)\,dx$

2. What do we get in the general case?



Clearly, $f(x)$ is negative on a portion of $[a,b]$.

$\int_a^b f(x)\,dx$ = Area − Area

= Net Area.

Example: Compute $\int_1^4 \left(x^2 - 3x\right)dx$. Then give a geometric interpretation for this integral.

$\int_1^4 (x^2 - 3x)\,dx = \left(\frac{1}{3}x^3 - \frac{3}{2}x^2\right)\Big|_1^4$

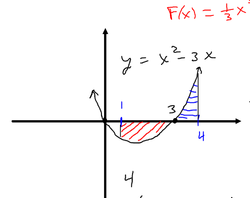
We need $F(x)$ so that $F'(x) = x^2 - 3x$

$F(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2$

$= \left(\frac{64}{3} - 24\right) - \left(\frac{1}{3} - \frac{3}{2}\right)$

$= 21 - 24 + \frac{3}{2}$

$= -\frac{3}{2}$.



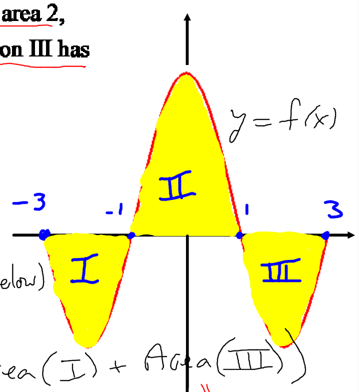
Note: $f(x) = x^2 - 3x$ is negative on a portion of $[1,4]$.

$\int_1^4 (x^2 - 3x)\,dx$ = Area − Area

(area above minus area below)

= Net Area.

Example: $y = f(x)$ is graphed on the right for $-3 \le x \le 3$. Region I has area 2, region II has area 3, and region III has area 2. Give $\int_{-3}^{3} f(x)\,dx$



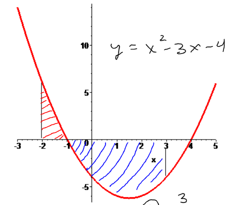
$$\int_{-3}^{3} f(x)\,dx = \text{Net Area}$$

$$= (\text{Area above}) - (\text{Area below})$$

$$= \underbrace{\text{Area}(\text{II})}_{=3} - \Big(\underbrace{\text{Area}(\text{I})}_{=2} + \underbrace{\text{Area}(\text{III})}_{=2}\Big)$$

$$= 3 - 4 \quad = -1.$$

Example: Give the area bounded between the x-axis and the curve $y = x^2 - 3x - 4$ over the interval $[-2, 3]$.



$y = x^2 - 3x - 4$

$\text{Area} = \text{Area}(\;) + \text{Area}(\;)$

Notes: ① $x^2 - 3x - 4 \ge 0$ on $[-2, -1]$.

$\therefore \text{Area}(\;) = \int_{-2}^{-1} (x^2 - 3x - 4)\,dx$

② $\int_{-1}^{3} (x^2 - 3x - 4)\,dx = 0 - \text{Area}(\;)$

$\therefore \text{Area}(\;) = -\int_{-1}^{3} (x^2 - 3x - 4)\,dx$

$$\Rightarrow \text{Area} = \int_{-2}^{-1} (x^2 - 3x - 4)\,dx - \int_{-1}^{3} (x^2 - 3x - 4)\,dx$$

$= \text{See the video}$

#3 $\quad 1 + 2 =$