Net Area and Integrals

- We are in chapter 5.
- New EMCFs and Homework will be posted.
- Test 3 runs through Monday.
- You have an online quiz and a practice test due Monday.
- No office hours today!!

Example: Compute $\int_0^2 (2x^2 - 3x - 3) \, dx$

We need $F(x)$

So $F(x) = \frac{3}{2} x^2 - \frac{3}{2} x^2 - 3x$

We get $F(x) = \left[ \frac{1}{3} x^3 - \frac{1}{2} x^3 - 3x \right]_0^2$

Riemann Integral of $f(x)$ on $[a, b]$.

Review: The first part of the Fundamental Theorem of Calculus

Theorem: If $f$ is a continuous function on the interval $[a, b]$, and $F$ is an antiderivative of $f$ then

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

Limits of integration.
Review:

The **Riemann** Integral of $f$ from $a$ to $b$ is

$$\int_a^b f(x) \, dx = \text{Area}$$

1. What do we get if $f$ is nonnegative on the interval $[a,b]$?

$$y = f(x) \quad \text{Note: } f(x) \geq 0 \text{ on } [a,b].$$

**Integrating:**

$$\int_a^b f(x) \, dx = \text{The area bounded between } y = f(x) \text{ and the } x \text{-axis for } x \text{ between } a \text{ and } b.$$  

2. What do we get in the general case?

$$y = f(x) \quad \text{Clearly, } f(x) \text{ is negative on a portion of } [a,b].$$

$$\int_a^b f(x) \, dx = \text{Area} - \text{Area} = \text{Net Area}.$$

**Example:** Give the area bounded between the $x$-axis and the graph of $y = x^3 + 1$ on the interval $[1,2]$.

**Not:** $f(x) = x^3 + 1$ is nonnegative on $[1,2]$.

**Area:**

$$\int_1^2 (x^3 + 1) \, dx = \left[\frac{x^4}{4} + x\right]_1^2 = \left(\frac{16}{4} + 2\right) - \left(\frac{1}{4} + 1\right) = 4 + 2 - \frac{5}{4} = \frac{9}{4}.$$

**Example:** Compute $\int (x^3 - 3x) \, dx$. Then give a geometric interpretation.

**Not:** $f(x) = x^3 - 3x$ is negative on a portion of $[1,4]$.

$$\int_1^4 (x^3 - 3x) \, dx = \left[\frac{x^4}{4} - \frac{3x^2}{2}\right]_1^4 = \left(\frac{256}{4} - 3 \cdot 16\right) - \left(\frac{1}{4} - 3 \cdot 1\right) = 21 - 24 + \frac{5}{4} = -\frac{7}{4}.$$
Example: \( y = f(x) \) is graphed on the right.

For \(-3 \leq x \leq 3\), region I has area 2, region II has area 3, and region III has area 2. Given \( \int_{-3}^{3} f(x) \, dx \)

\[
\int_{-3}^{3} f(x) \, dx = \text{Net Area} \\
= \text{(Area above)} - \text{(Area below)} \\
= \text{Area (II)} - (\text{Area (I)} + \text{Area (III)}) \\
= \frac{3}{2} - \frac{1}{2} = -1.
\]