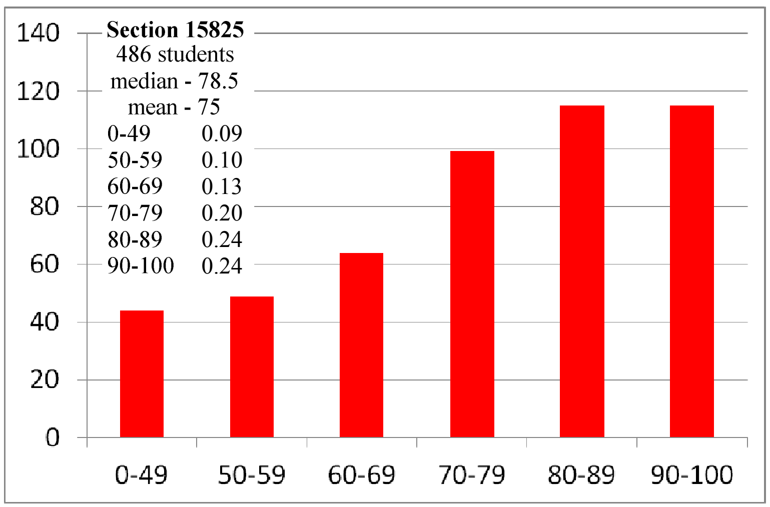
Net Area and Integrals

- We are in chapter 5.
- I posted a video with some upper sum and lower sum examples.
- The written portion of Test 3 is graded. Add the Test 3 and Test 3 FR columns in the gradebook to obtain your grade.

Grade Information by Section

15819		15825		15836		15841	
503 students		486 students		324 students		121 students	
median - 69		median - 78.5		median - 71		median - 66	
mean - 67.5		mean - 75		mean - 69.6		mean - 63.6	
0-49	0.17	0-49	0.09	0-49	0.16	0-49	0.25
50-59	0.15	50-59	0.10	50-59	0.13	50-59	0.13
60-69	0.18	60-69	0.13	60-69	0.18	60-69	0.19
70-79	0.18	70-79	0.20	70-79	0.18	70-79	0.20
80-89	0.18	80-89	0.24	80-89	0.18	80-89	0.16
90-100	0.12	90-100	0.24	90-100	0.17	90-100	0.07

Grade Distribution - This Class



Extra Credit for Test 3

You can earn 5 additional points on Test 3 by going to

http://www.casa.uh.edu/TeacherEvaluation/

Fill out the teacher evaluation for both your lecture and lab section.

Upon completion, I will receive a note in my inbox with your student ID. Your teacher evaluation WILL NOT be forwarded with your ID. The information you give is anonymous.

http://www.casa.uh.edu/TeacherEvaluation/

Department of Mathematics

Online Teacher Evaluation Forms

! Disclaimer Notice !

The following information must be provided in order to access your course evaluation form(s). Your First Name, Last Name and PeopleSoft ID are confidential. They will only be used to determine which class(es) you are enrolled in. This information will not be passed along to the course instructor.

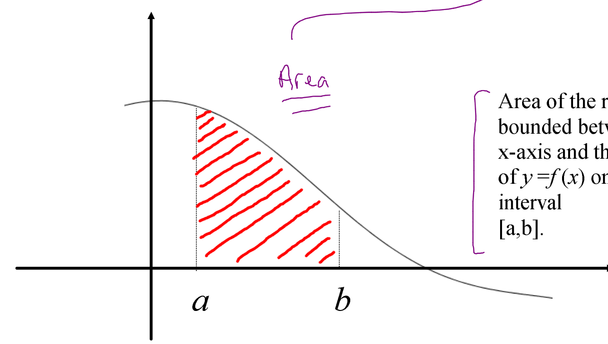
Firstname:	
Lastname:	
PeopleSoft ID:	

Login

Recall:

The **Riemann** Integral of f from a to b: $\boxed{\int_a^b f(x)dx}$

1. What do we get if $\boxed{f \text{ is nonnegative}}$ on the interval $[a,b]$?

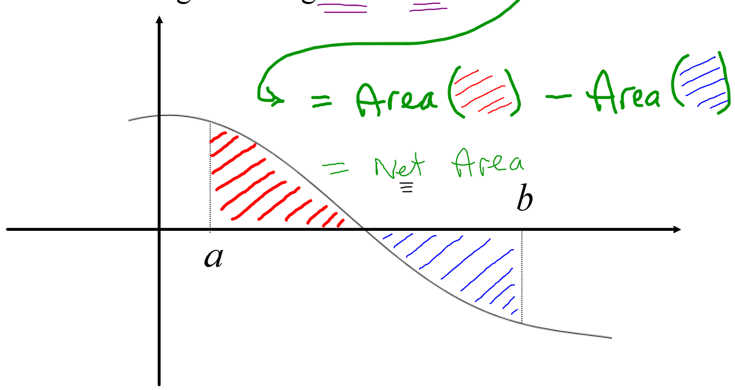


Area of the region bounded between the x-axis and the graph of $y = f(x)$ on the interval [a,b].

Recall:

The **Riemann** Integral of f from a to b: $\int_a^b f(x)dx$

2. What do we get in the general case?



Popper P25

$\int_{-1}^{5}(x^2-3x-4)\,dx$

1. Find the Riemann integral of $f(x) = x^2 - 3x - 4$ on the interval [-1,5].

2. Find the area bounded between the x-axis and the graph of $f(x) = x^2 - 3x - 4$ on the interval [-1,5].

Properties of the Riemann Integral:

1. $\int_a^b \left(f(x)+g(x)\right)dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

2. $\int_a^b \alpha f(x)\,dx = \alpha \int_a^b f(x)\,dx$, α is a real number.

3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

4. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

5. $\int_a^a f(x)\,dx = 0$

6. If $f(x) \ge g(x)$ on $[a,b]$, then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$.

Example: Suppose $\int_0^1 f(t)dt = 2$ and $\int_0^3 f(t)dt = -4$. Give

$$\int_1^3 3f(t)dt. = 3\int_1^3 f(t)\,dt$$

$\int_a^b \alpha f(x)\,dx = \alpha \int_a^b f(x)\,dx$, α is a real number.

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$$\underset{\substack{\parallel \\ -4}}{\int_0^3 f(t)\,dt} = \underset{\substack{\parallel \\ 2}}{\int_0^1 f(t)\,dt} + \int_1^3 f(t)\,dt$$

$$\therefore \int_1^3 f(t)dt = -6 \Rightarrow \int_1^3 3f(t)dt = 3\int_1^3 f(t)dt$$

$$= 3(-6)$$

$$= -18.$$

Recall: The first part of the **Fundamental Theorem of Calculus**

Theorem: If f is a continuous function on the interval $[a,b]$, and F is an anti derivative of f, then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

New: The second part of the **Fundamental Theorem of Calculus**

Theorem : If f is a continuous function, then

$$\frac{d}{dx}\boxed{\int_a^x f(t)dt} = f(x)$$

a = constant

x = variable

Function of x

Why? If we have an anti derivative F for f then

$$\int_a^x f(t)\,dt = F(t)\Big|_a^x = F(x) - \underbrace{F(a)}_{\text{number}}$$

$$\therefore \frac{d}{dx}\int_a^x f(t)dt = \frac{d}{dx}\left[F(x) - F(a)\right] = F'(x)$$

$$= f(x)$$

Examples: $\frac{d}{dx}\int_0^x \sin(t)\,dt = \sin(x)$

$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$

Note: $\int_0^x \sin(t)\,dt = -\cos(t)\Big|_0^x = -\cos(x)+1$

So $\frac{d}{dx}\int_0^x \sin(t)\,dt = \frac{d}{dx}(-\cos(x)+1) = \sin(x)$

$\frac{d}{dx}\int_0^x \frac{1}{\sin(t)+2}\,dt = \frac{1}{\sin(x)+2}$

$\underbrace{\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt = \frac{d}{du}\int_0^u \sin(t)\,dt \cdot \frac{du}{dx}}_{\text{chain rule}}$ ($x^2 = u(x)$)

$= \sin(u)\cdot\frac{du}{dx}$

$= \sin(x^2)\cdot 2x$

$= 2x\sin(x^2)$

New: The final part of the **Fundamental Theorem of Calculus**

Theorem : If f is a continuous function and u is a differentiable function, then

$$\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f(u(x))\,u'(x)$$

$a =$ constant

$$\| $$

$$\frac{d}{du}\int_a^u f(t)\,dt \cdot \frac{du}{dx}$$

Examples: $\frac{d}{dx}\int_0^{2x} \sin^2(t)\,dt = \sin^2(2x)\cdot 2$ ($2x = u(x)$)

$= 2\sin^2(2x)$

$\boxed{\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f(u(x))\,u'(x)}$

$\frac{d}{dx}\int_{3x}^0 \frac{1}{\sin(t)+2}\,dt = \frac{d}{dx}\left[-\int_0^{3x}\frac{1}{\sin(t)+2}\,dt\right]$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$= -\frac{d}{dx}\int_0^{3x}\frac{1}{\sin(t)+2}\,dt = -\frac{1}{\sin(3x)+2}\cdot 3$

$= \frac{-3}{\sin(3x)+2}$

$\frac{d}{dx}\int_{2x}^{x^2} \frac{1}{\sqrt{t}+\cos(t)+3}\,dt =$

#3 1/2

#4 6

$= \frac{d}{dx}\left[\int_{2x}^{1}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt + \int_1^{x^2}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt\right]$

$= -\frac{d}{dx}\int_1^{2x}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt + \frac{d}{dx}\int_1^{x^2}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt$

$= -\frac{2}{\sqrt{2x}+\cos(2x)+3} + \frac{2x}{\sqrt{x^2}+\cos(x^2)+3}$

Example: Find a function f so that $2x+\cos(3x) = \int_x^1 f(t)\,dt$

See the video.