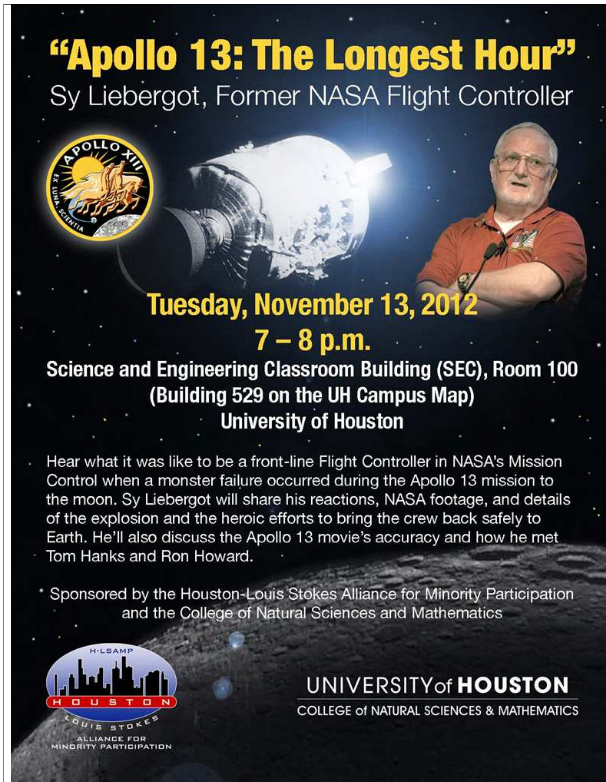
Info...

- A **Quiz** is due tonight.
- A new **EMCFs** and **Homework** are posted.

Sy Liebergot, Former NASA Flight Controller

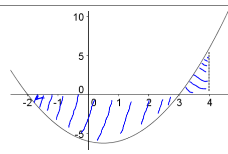
"Apollo13: The Longest Hour"

Tuesday, November 13
7 – 8 pm in SEC 100



Popper P27

1. $\int_{-2}^{4}\left(x^2 - x - 6\right)dx =$



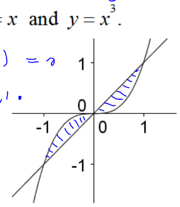
2. Find the area bounded between the x-axis and the graph of $y = x^2 - x - 6$ on the interval $[-2,4]$.

$-\int_{-2}^{3}(x^2 - x - 6)dx + \int_{3}^{4}(x^2 - x - 6)dx$

3. Find the are bounded between the curves $y = x$ and $y = x^3$.

$x^3 = x \qquad x^3 - x = 0 \qquad x(x^2 - 1) = 0$

$x = -1, 0, 1.$



Note: The Area btwn x and x^3 on $[-1, 0]$ is the same as the area btwn x and x^3 on $[0, 1]$.

So, Area $= 2\int_0^1 (\text{Top} - \text{Bottom})dx = 2\int_0^1 (x - x^3)dx$

$= 2\left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Big|_0^1 = 2\left[\left(\frac{1}{2} - \frac{1}{4}\right) - (0)\right] = 1/2.$

No limits of integration → $\int f(x)dx = F(x) + C$ → An anti-deriv. + Constant

$\int f(x)dx$ = "The **general anti-derivative** of f."

"**The indefinite integral of f.**"

"The integral of f."

Any 2 anti-deriv. differ by a constant.

Note: Spse $f(x)$ is continuous. Spse $F(x)$ and $G(x)$ are anti-derivatives of $f(x)$.

Define $H(x) = F(x) - G(x)$.

Pick an interval $[a, b]$ where this is defined. MVThm ⇒ there is a value c btwn a and b so that $H'(c) = \frac{H(b) - H(a)}{b - a}$.

$H'(c) = F'(c) - G'(c) = f(c) - f(c) = 0$

∴ $H(b) = H(a)$.

i.e. since a and b are arbitrary $H(x) \equiv$ constant.

Examples: $\int x^2 dx = \frac{1}{3}x^3 + C$ ← arbitrary constant

$\int \cos(x)dx = \sin(x) + C$

$\frac{d}{dx}\left(-\frac{1}{2}\cos(2x)\right) = \frac{1}{2}\sin(2x)\cdot 2$

$\int \sin(2x)dx = -\frac{1}{2}\cos(2x) + C$

$\int\left(-x^2+\sqrt{x}-\frac{3}{x^2}\right)dx = \int\left(-x^2 + x^{1/2} - 3x^{-2}\right)dx$

$= -\frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + 3x^{-1} + C$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$

$n \neq -1$

Note: Most differentiation involves the chain rule, so we should expect that most antidifferentiation will involve

undoing the chain rule.

(*) (u-substitution)

Undoing the chain rule...

$F'(x)$	$F(x)$
$2\cos(2x)$	$\sin(2x) + C$
$2x\cos(x^2)$	$\sin(x^2) + C$
(*) $\cos(u)\frac{du}{dx}$	$\sin(u) + C$
$8x(x^2+1)^3$	$(x^2+1)^4 + C$
$(n+1)u^n\frac{du}{dx}$	$u^{n+1} + C$ $(n \neq -1)$
(*) $u^n\frac{du}{dx}$	$\frac{1}{n+1}u^{n+1} + C$
(*) $\sin(u)\frac{du}{dx}$	$-\cos(u) + C$

Undoing the chain rule...

$\int u^n du = \frac{1}{n+1}u^{n+1} + C$, $n \neq -1$

$\int \cos(u)du = \sin(u) + C$

$\int \sin(u)du = -\cos(u) + C$

$\int \sec^2(u)du = \tan(u) + C$

others?

$\int \csc^2(u)du = -\cot(u) + C$

Examples: $\int \cos(2x)2dx = \sin(2x) + C$

$u = x^2$, $du = 2x\,dx$

$\int x\sin(x^2)dx = \frac{1}{2}\int \sin(x^2)\,2x\,dx = \frac{1}{2}\int \sin(u)\,du$

$= -\frac{1}{2}\cos(u) + C = -\frac{1}{2}\cos(x^2) + C$

$u = x^2+1$, $du = 2x\,dx$

$\int x(x^2+1)^4 dx = \frac{1}{2}\int (x^2+1)^4\,2x\,dx = \frac{1}{2}\int u^4 du$

$= \frac{1}{10}u^5 + C = \frac{1}{10}(x^2+1)^5 + C$

$u = 2+\sin(x)$, $du = \cos(x)dx$

$\int \frac{\cos(x)}{\sqrt{2+\sin(x)}}dx = \int (2+\sin(x))^{-\frac{1}{2}}\cos(x)dx$

$= \int u^{-\frac{1}{2}}du = 2u^{1/2} + C$

$= 2\sqrt{2+\sin(x)} + C$

Question: How do we handle u-substitution with a definite integral?

Answer: We change the limits of integration to reflect the substitution.

Example: $\int_0^1 3x^2\sin(2x^3-1)dx =$

$u = 2x^3 - 1$

$du = 6x^2 dx$

$x = 1 \Rightarrow u = 1$

$x = 0 \Rightarrow u = -1$

$= \frac{1}{2}\int_{-1}^{1} \sin(u)\,du$

$= \frac{1}{2}\int_{-1}^{1} \sin(u)\,du$

$= -\frac{1}{2}\cos(u)\Big|_{-1}^{1}$

$= -\frac{1}{2}(\cos(1) - \cos(-1))$

$= 0$

Note: $\cos(1) = \cos(-1)$ b/c Cosine is even.

Example: $\int_{-2}^{1} x(x^2+1)^4 dx =$

See the video!!