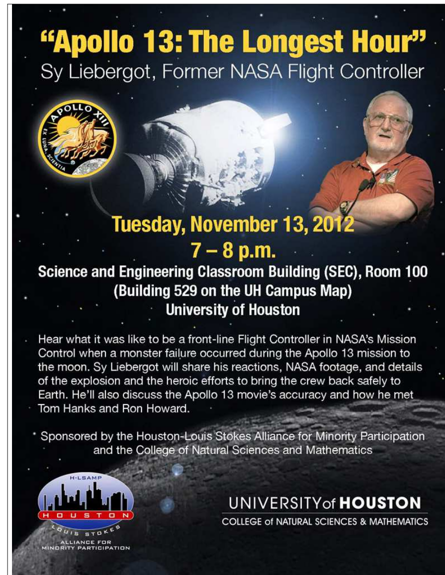
Info...

- A **Quiz** is due tonight.
- A new **EMCFs** and **Homework** are posted.

Sy Liebergot, Former NASA Flight Controller

"Apollo13: The Longest Hour"

Tuesday, November 13
7 – 8 pm in SEC 100



$\int f(x)dx$ = **The general anti-derivative of f.**

(The $\boxed{\text{indefinite integral}}$ of f.)

Function + C

arbitrary constant.

Examples:

$\int x^2 dx = \frac{x^3}{3} + C$ arbitrary constant

$\int \cos(x) dx = \sin(x) + C$

$\int \sin(2x) dx = -\frac{1}{2}\cos(2x) + C$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1.$

$\int \left(-x^2 + \sqrt{x} - \frac{3}{x^2}\right) dx = \int (-x^2 + x^{1/2} - 3x^{-2}) dx$

$= -\frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + 3x^{-1} + C$

Note: Most differentiation involves the chain rule, so we should expect that most antidifferentiation will involve

$\boxed{\textbf{undoing the chain rule.}}$

(u-substitution)

Undoing the chain rule...

$F'(x)$	$F(x)$
$2\sin(2x)$	$-\cos(2x)+C$
$3\sin(2x)$	$-\frac{3}{2}\cos(2x)+C$
$\frac{2x}{\sqrt{x^2+1}} = 2x(x^2+1)^{-\frac{1}{2}}$	$2(x^2+1)^{1/2}+C$
$u^n\frac{du}{dx}$	$\frac{1}{n+1}u^{n+1}+C$
$\sin(u)\frac{du}{dx}$	$-\cos(u)+C$

Undoing the chain rule...

$$\int u^n du = \frac{1}{n+1}u^{n+1}+C \quad , \quad n \neq -1$$

$$\int \cos(u)\,du = \sin(u)+C$$

$$\int \sin(u)du = -\cos(u)+C$$

$$\int \sec^2(u)du = \tan(u)+C$$

others? $\int \csc^2(u)du = -\cot(u)+C$

Examples:

$$\int \underbrace{\cos(2x)2dx} = \sin(2x)+C$$

$u = x^2$

$du = 2x\,dx$

$$\int x\sin(x^2)dx = \frac{1}{2}\int \sin(2x)\underbrace{2x\,dx} = \frac{1}{2}\int \sin(u)\,du$$

$$= -\frac{1}{2}\cos(u)+C$$

$$= -\frac{1}{2}\cos(x^2)+C$$

$$\int x(x^2+1)^4 dx =$$

$$\int \frac{\cos(x)}{\sqrt{2+\sin(x)}}dx =$$

$u = x^2+1$

$du = 2x\,dx$

$$\int x\left(x^2+1\right)^4 dx = \tfrac{1}{2}\int (x^2+1)^4\, 2x\,dx = \tfrac{1}{2}\int u^4\,du$$

$$= \tfrac{1}{2}\cdot\tfrac{1}{5}u^5 + C_1 = \tfrac{1}{10}(x^2+1)^5 + C$$

$u = 2+\sin(x)$

$du = \cos(x)\,dx$

$$\int \frac{\cos(x)}{\sqrt{2+\sin(x)}}\,dx = \int \frac{1}{\sqrt{u}}\,du$$

$$= \int u^{-\frac{1}{2}}\,du$$

$$= 2u^{1/2} + C$$

$$= 2\sqrt{2+\sin(x)} + C$$

Question: How do we handle u-substitution with a definite integral?

Answer: We change the limits of integration to reflect the substitution.

Example:
$$\int_0^1 3x^2 \sin\left(2x^3-1\right)dx = \tfrac{1}{2}\int_0^1 \sin(2x^3-1)\,2\cdot 3x^2\,dx$$

$u = 2x^3-1$

$du = 6x^2\,dx$

$x=1 \Rightarrow u=1$

$x=0 \Rightarrow u=-1$

$$= \tfrac{1}{2}\int_{\boxed{-1}}^{\boxed{1}} \sin(u)\,du$$

$$= \tfrac{1}{2}\left(-\cos(u)\right)\Big|_{-1}^{1}$$

$$= \tfrac{1}{2}\left[\left(-\cos(1)\right) - \left(-\cos(-1)\right)\right]$$

$$= \tfrac{1}{2}\left(-\cos(1) + \cos(1)\right) = 0.$$

Cosine is an even function

$\therefore \cos(-1) = \cos(1)$

Example:
$$\int_{-2}^{1} x\left(x^2+1\right)^4 dx = \tfrac{1}{2}\int_{-2}^{1} (x^2+1)^4\, 2x\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$x=1 \Rightarrow u=2$

$x=-2 \Rightarrow u=5$

$$= \tfrac{1}{2}\int_{\boxed{5}}^{\boxed{2}} u^4\,du$$

$$= \tfrac{1}{2}\cdot\tfrac{1}{5}u^5\Big|_5^2$$

$$= \tfrac{1}{10}\left(32 - 5^5\right) = \text{you.}$$